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Skin-Effect Resistance Measurements of Conductors at Radio-Frequencies up to 100 000 Cycles per Second

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A. E. KENNELLY and H. A. AFFEL

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Electrical Engineering Department
Massachusetts Institute of Technology
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SKIN-EFFECT RESISTANCE MEASUREMENTS OF CONDUCTORS

At Radio-Frequencies up to 100,000 Cycles Per Second*

By

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and

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The research here reported was conducted in the Research Division, of the Electrical Engineering Department of the Massachusetts Institute of Technology, during 1915–16, under an appropriation from the American Telephone and Telegraph Company. The object of the research was to obtain experimental data on the alternating-current resistances of various electrical conductors at frequencies up to 100,000 $\sim$.

OUTLINE OF PRECEDING HIGH-FREQUENCY ALTERNATING-CURRENT RESISTANCE MEASUREMENTS

Publications of skin-effect measurements at high frequencies are very rare. The Bibliography of the Kennelly-Laws-Pierce A. I. E. E. paper of 1915, and that appended to this paper, contain references to the work of Fleming, Lindemann, Dolezalek and others, in this direction. Fleming's measurements were published in 1910, and he was able to obtain confirmation of Kelvin's formula for the skin-effect resistance ratio of certain straight wires using oscillatory currents. The measurements here reported seem to differ from those previously published, in that they have been obtained with steadily sustained alternating currents, furnished by an a. c. generator, and using a null method, with a somewhat sensitive induction bridge. The research

*Presented as the Presidential Address before The Institute of Radio Engineers, New York, May 3, 1916. Manuscript received April 4, 1916. A table of the symbols used is given at the end of the paper.

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constitutes a continuation, at higher frequencies, of that reported in the A. I. E. E. 1915 paper above mentioned.

In that paper it was reported that the standard skin-effect formulas for round wires had been observed to hold, within the limits of experimental error, up to 5,000~, the highest frequency used. It was also noted that the skin-effect resistance ratio of conductors formed of 7 round bare strands, with normal spirality, was slightly greater than that of the equisectional solid wire. Straight flat-strip conductors were found to have a skin-effect resistance ratio far in excess of that predicated from existing available formulas.

The ratio Z/R of the linear a. c. internal impedance of a conductor (Z ohms $\angle$ per centimeter), to the linear d. c. resistance (R ohms per centimeter), is a complex numeric called the *skin-effect impedance ratio.*[1] Its real component R'/R, is called the *skin-effect resistance ratio*, and is the usual factor which expresses the magnitude of skin-effect. The imaginary component X/R, is similarly called the *skin-effect reactance ratio.* This paper deals almost exclusively with the skin-effect resistance ratio R'/R. When subdivided wires are cabled, they are spiralled for mechanical reasons. This usually increases the resistance ratio, owing to what is called the *spirality effect.* When wires carrying alternating currents are placed in such a degree of proximity as to affect appreciably the distribution of alternating-current density over their cross section, owing to their mutually interacting magnetic fields, the resistance ratio R'/R is ordinarily increased, owing to *proximity effect.* The ordinary skin-effect resistance ratio of a long straight conductor is therefore to be understood as occasioned by the distortion of alternating-current density over its cross section, due entirely to the magnetic field of that conductor. This research has been directed to the measurement of the resistance ratio in straight conductors only, in view of the fundamental character of such measurements, and the sparsity of published observations in this field.

GENERATING APPARATUS

The source of the alternating currents used in all the tests was an Alexanderson radio-frequency[2] alternator made by the General Electric Co., and kindly loaned to our Department, by the National Electric Signaling Co. and Professor R. A. Fes-

[1]The angle sign $\angle$ indicates a complex quantity or "plane vector."

[2]Radio frequencies are, for convenience, arbitrarily chosen as those over 10,000 cycles per second.

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senden. A picture of the machine appears in Figure 1. The direct-current driving motor *M*, on the right-hand side, is a 4-pole 125-volt 82-ampere machine, operating, when at full speed, at 2,000 r. p. m., and driving the alternator on the left,

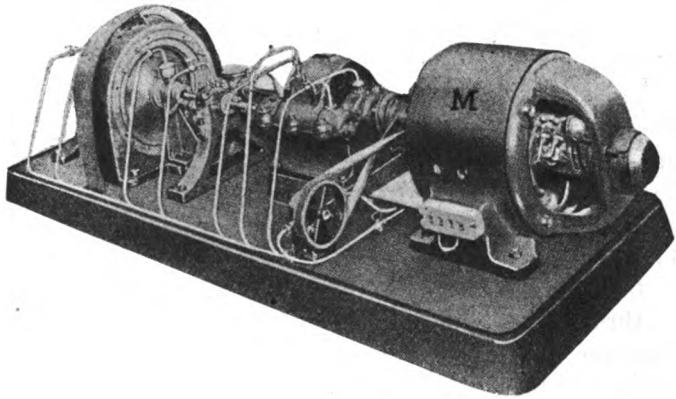

FIGURE 1—Motor-Driven High-Frequency Generator

thru a 10:1 step-up gearing, at a speed of 20,000 r. p. m. The alternator, of the inductor type, has a steel disk rotor, represented in Figure 2, with 300 radial slots. The stationary armature

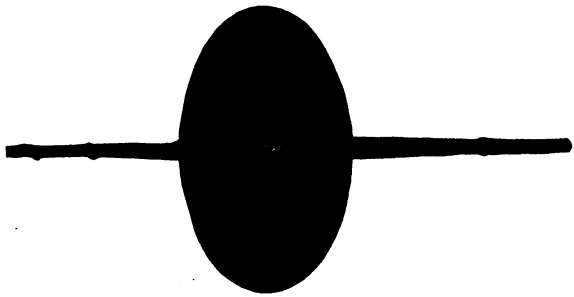

FIGURE 2—Rotor and Shaft of Standard 100,000-Cycle Alternator

windings are supported on polar projections, on each side of the rotor, so that there is one complete cycle of induced e. m. f. in the armature winding at the passage of each tooth on the rotor. The full-speed frequency is therefore $20,000 \times 300 = 6,000,000$ cycles per minute, or 100,000 cycles per second. The

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alternator is so designed that the clearance between the rotor and the stator pole faces can be closely adjusted. With a small air gap, power up to 2 kilowatts, at about 15 amperes and 130 volts, can be obtained. In the tests described, the current taken from the machine rarely exceeded 5 amperes, thus enabling a less troublesome mechanical clearance adjustment to be used. The machine, which is self-oiling by forced lubrication, could be run at constant full speed for an hour or more without particular attention.

The wave form of the generated e. m. f. was not investigated, but no evidence has presented itself of any appreciable irregularities, to which moreover the additional reactance automatically developed would be enormous.

The generator set was placed in a room about 20 meters away from the testing laboratory, and controlled from the latter, thru a set of armature and field resistances. A calibrated little magneto generator, geared to the motor shaft, but not shown in the picture, actuates a calibrated d. c. voltmeter on the testing table, so as to indicate continuously both speed and frequency.

METHODS OF MEASUREMENT

The first means for measurement tried, consisted of a simple Ohm's Law circuit, illustrated in Figure 3, using a hot-wire voltmeter V, and ammeter A. A series circuit was made up, comprising the alternator G, a hot-wire ammeter A, the test conductor X, and the adjustable condenser C. This condenser was needed in measuring the resistance of the test-wire X, in order to compensate for the relatively high reactance, that even a simple grid distribution of a few meters of test conductor will offer at so high a frequency. The operation consists in causing the radio-frequency current to flow in the circuit, and so adjusting the variable condenser C, that the minimum e. m. f. is indicated by the hot-wire voltmeter V, connected as shown. Under these circumstances, the ratio E/I of the branch circuit $a\ b\ c$, will be the effective a. c. resistance of the test conductor and condenser, the latter being relatively small.

The method has the advantages of simplicity and directness of measurement. It is swift and convenient. It has the disadvantages that it is not a null method, and that the final resonance adjustment is very troublesome; also, that at least two ranges of voltmeter reading are required, in order to complete the adjustment. At resonance; i. e., when the inductive reactance of the branch circuit $a\ b\ c$, is equal and opposite to

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the reactance of the condenser C, the current thru the ammeter A is relatively large; while the e. m. f. indicated on the voltmeter V is relatively small. The capacitance C is such that for low-resistance test wires, at frequencies not exceeding 100,000 ~, an air condenser cannot be used, and hence the apparent resistance of the test wire is increased, owing to the

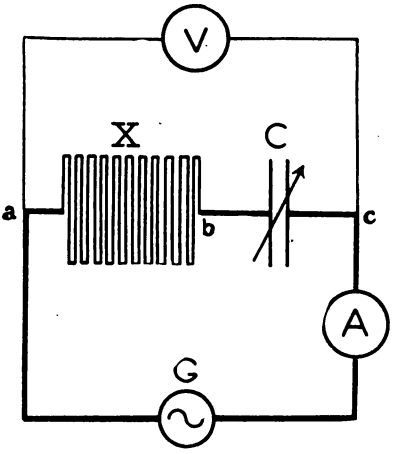

FIGURE 3—Radio-Frequency Resistance Measurement by Ohm's Law Circuit

presence of power loss in condensers with solid dielectrics. For these reasons, the method was later discarded, in favor of an inductive-bridge method; but it is nevertheless recommended as a suitable method in commencing the study of high-frequency resistances; besides serving as a convenient check upon such null methods as may later be employed.

The test wires employed had apparent radio-frequency resistances of less than half an ohm. The condenser C was adjustable up to a maximum of 26 μf. (microfarads), by steps of 0.001 μf., and a final adjustment, with the aid of a little rotary oil-condenser, continuously variable to a total of about 4 mμf. (millimicrofarads).

There were two multiple-scale Hartmann and Braun voltmeters available, covering a range from 0 to 200 volts, and a multiple-scale ammeter, covering a total range from 0 to 5 amperes without shunt. Altho calibrated with continuous currents, the design and construction of these instruments warrants the belief that their calibration is substantially correct for alternating currents over the range of frequency employed.

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INDUCTIVE BRIDGE METHOD

The bridge method later employed corresponds, in essential features, to that described by A. Hund* in 1915. The diagrammatic connections are indicated in Figure 4. A is the radio-frequency alternator, and X is the conductor under test. It is balanced, as to resistance and inductance, by the adjustable resistor R, and the adjustable inductor L. The equal arms of

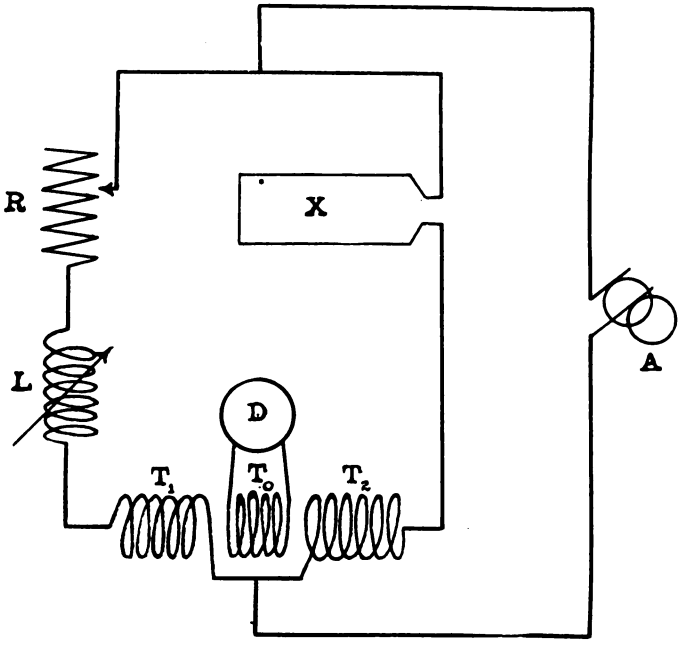

FIGURE 4—Differential Transformer Bridge System

the bridge are the two opposing primary coils T_1 and T_2 of a small special testing transformer, the secondary winding T_o of which is connected to the detector D. When zero balance is obtained in the detector D, for any measured impressed frequency within the range of the alternator A, the values of R and L are respectively the apparent resistance and inductance of the test conductor X, corresponding to that frequency.

The details of the apparatus, as used in practice, are shown in Figures 27 and 28, the description of which is given in Appendix I, in such detail as should enable the apparatus to be duplicated if desired.

*Bibliography, Number 14.

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CONDUCTORS TESTED

Straight Solid Wires.—A round solid annealed copper wire number 12 A. W. G. (diameter 0.205 cm. = 0.0808 inch) was tested by both of the above methods. The results are given in Figure 5. The curve is the locus of the skin-effect resistance

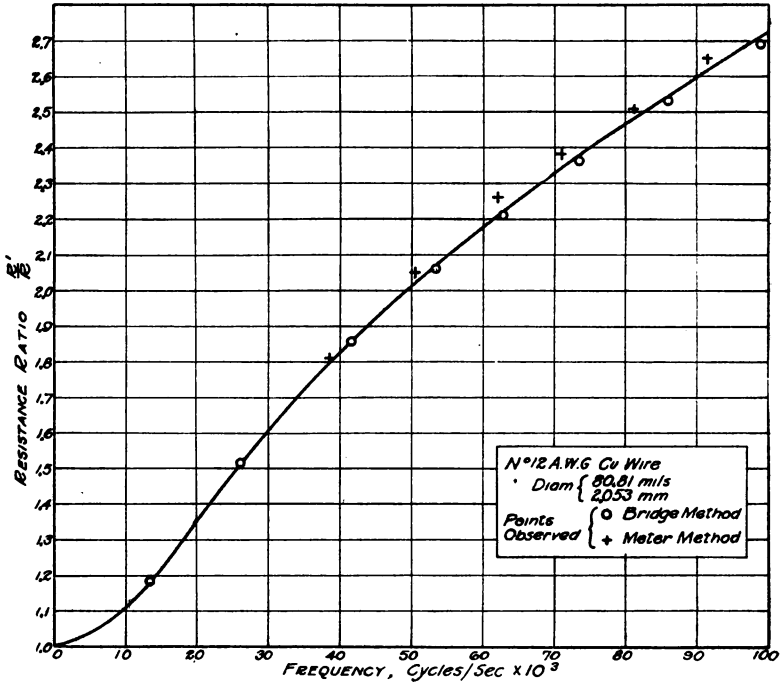
RESISTANCE RATIO R'/R
FREQUENCY, Cycles/Sec x 10³
N°12 A.W.G Cu Wire
Diam { 80.81 mils 2.053 mm
Points Observed { Bridge Method Meter Method

FIGURE 5—Skin Effect Ratio for Number 12 A.W.G. Copper Wire as Observed and Computed

ratio R'/R, as computed from the real part of the Bessel-function formula:

$$\frac{Z}{R} = \frac{a_o X}{2} \cdot \frac{J_o(a_o X)}{J_1(a_o X)} \qquad \text{numeric} \angle \quad (1)$$

where Z is the internal linear impedance of the wire, of which R' is the linear resistance component, X is the radius of the wire in centimeters, and $a_o = \sqrt{-j\,4\pi\gamma\mu\omega}$ (see Appendix II). The crosses represent observations by the Ohm's law method, and the circles by differential method. It will be seen that the latter observations are in substantial accordance with the computed curve. The wire was mounted over insulators, on a flat frame

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illustrated in Figure 6, which was made standard in these tests. The advantage of this particular mounting is that it holds the wires in one plane at a uniform spacing (about 5 cm. or 2 inches), sufficiently great to make the proximity effect negligibly small.

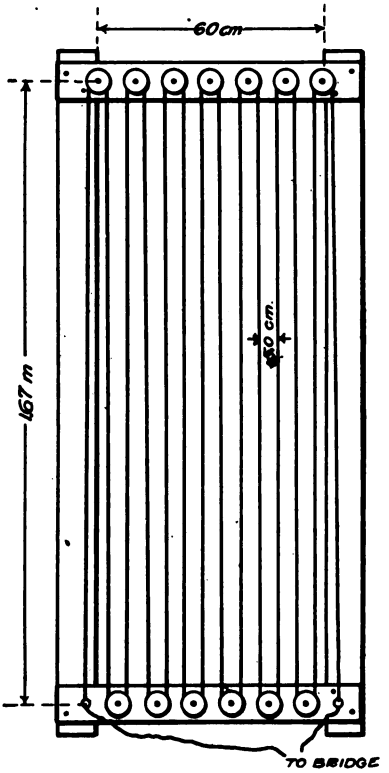

FIGURE 6—Test Wire on Frame

The external inductance of the wire, thus mounted, is also relatively small. It was ascertained that altering the spacing of the conductor on the frame; even to the extent of placing the 25 meters of wire in a single horizontal loop, 20 centimeters wide, on the wall, did not appreciably affect the skin-effect resistance ratio R'/R, to the degree of precision used in these tests. The mounting of the test wire on such a frame has the additional advantage that it brings the wire into a small compass, and minimizes its external electromagnetic disturbing fields on the measuring apparatus. Moreover, it enables a framed test wire to be kept indefinitely for reference, when desired.

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A second test wire, formed of 25.3 meters of number 14 A. W. G. solid round copper wire (diameter 1.625 mm. = 0.064 inch) was tested by differential bridge at 22° C., at a later period, and with greater precision. Its linear resistance was found to be 0.00825 ohm per meter. The skin-effect resistance ratio of this wire was found to agree with the value computed from the standard formula, within the limits of observational error, the following results being obtained:

Frequency Cycles/second	Skin-effect Resistance Ratio R'/R		Ratio Observed / Computed
	Observed	Computed	
96,100	2.192	2.189	1.002
82,200	2.051	2.049	1.001
59,300	1.775	1.774	1.001
38,000	1.453	1.458	0.997
20,000	1.162	1.170	0.993

The entire series of observations are presented by the circles in Figure 7, the curve showing the Bessel-function computed values, according to formula (1). It will be seen that there is a satisfactory agreement between the observations and this curve. Altho no reason can perhaps be assigned why the skin-effect resistance ratio of a round wire should depart, at greater frequencies, from the theoretical formula, already found to hold up to 5000 ∞; yet it is satisfying to find the agreement maintained. It should be noted that when attempts are made to measure with precision the skin-effect resistance ratio of such a straight round wire, the temperature of the wire must be carefully noted.*

STRANDED WIRES

Stranded Copper Wires.—After ascertaining that round solid copper wires, at radio frequencies, conformed to standard formulas, measurements were extended to subdivided or stranded copper wires with insulated strands. A test was made of stranded wire, of 20 strands, each number 26 A. W. G. (0.405 mm.) and enamel insulated to a diameter of 0.42 millimeter. The strands had a usual amount of spiralling or lay (4 cm. pitch). The unbraided diameter, i. e., the diameter of the bundle of strands,

*The result is also in conformity with that obtained by Fleming, thru another method, to frequencies up to 900,000 ∞. See Bibliography, Number 3.

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under the braiding, was about 25 per cent. greater than that of an equisectional solid or unstranded wire. The skin-effect resistance ratio R'/R, for this stranded wire, between 50,000 ∿ and 100,000 ∿, was about 10 per cent. less than that computed for the equisectional solid wire.

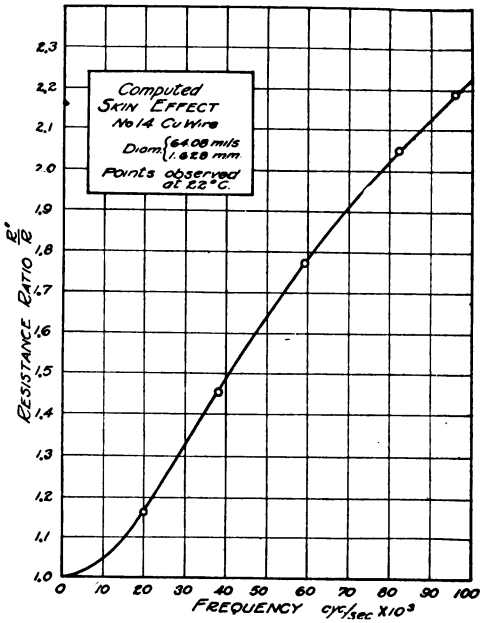

FIGURE 7—Agreement between Measured and Computed Skin-Effect Ratios for Round Wire

Another stranded wire had 50 strands of number 28 A. W. G. copper wire (0.32 mm., silk covered to 0.45 mm.) the strands being made up with a very long lay (about 1 turn per meter) and covered with braiding. The unbraided diameter was about 80 per cent. greater than that of the equisectional solid wire. This conductor had a skin-effect resistance ratio, at 100,000 ∿, about 33 per cent. less than that computed for the equisectional solid wire. Since measurements of the skin-effect resistance ratio of seven-strand copper cables, last year, showed a slight increase over the equisectional solid conductor, at frequencies up to 5,000 ∿, as stated in the above-mentioned A. I. E. E. paper of August, 1915, it seemed desirable to separate the effects of subdivision thru insulation, from those of spirality. By the courtesy of the Simplex Wire and Cable Co. of Boston, we were

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able to secure a sample of 26 meters of a stranded and braided wire, containing 48 strands, each of number 30 A. W. G. copper wire, diameter 0.255 millimeter, silk insulated to about 0.33 millimeter. These strands were braided up without twist, by being carefully drawn thru a die, whipped with silk thread by hand, and then laid on the supporting wooden frame for testing. By this treatment, it was hoped and believed that each individual strand would preserve its proper position substantially unchanged in the cross-section thruout the sample. The over-all unbraided diameter was about 50 per cent. greater than that of the equisectional solid wire. Another sample of stranded wire was made up at the same time, except that it had the usual amount of spirality, i. e., 1 turn in 4 centimeters about the central axis, and was machine braided.

Figure 8 shows the results of the skin-effect measurements on these two stranded conductors, and gives the resistance ratio R'/R against impressed frequency, up to 100,000 $\sim$. The curve A refers to the stranded untwisted sample, B to the stranded twisted sample, and C to the computed solid equisectional conductor. It will be seen that the subdivision of the wire into separate parallel untwisted strands, with this amount of silk separation, reduced the resistance ratio, at 100,000 $\sim$, from 2.38 to 1.86; but that the spirality effect of the lay in sample B, brought the ratio nearly half way back, or to 2.11. The B curve lies roughly midway between the A and C curves thruout. The reasons for the greater skin effect in the B sample over that of the A sample are probably twofold; namely (1), the additional copper losses in B due to the spirality effect; i. e., the longitudinal component of alternating magnetic field in the interior of the wire B, due to the action of the lay as a solenoid, and (2) the greater tightness under the braiding of the spiralled B sample, than that which could be safely given to the unspiralled A sample by hand, with a corresponding diameter enlargement ratio of only 1.4 for B, as against 1.5 for A.

If there were no insulation separating the individual parallel strands in the A sample, then it is believed that the skin-effect resistance ratio would rise into coincidence with C, or would become the same as in the equisectional solid wire. In other words, the theory and tests, to date, of skin effect in round wires, seem to show that the mere subdivision of the wire into a round bundle of parallel round filaments, which make contact with each other, but entrain channeled air spaces, has no appreciable effect on the skin-effect ratio; because the magnetic

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flux maintains a cylindrical distribution. When, however, even a thin layer of insulation separates the individual strands, some magnetic flux circulates radially to the mass, around each strand, and diminishes the skin-effect resistance ratio of the whole, while raising somewhat the skin-effect reactance ratio of the whole.

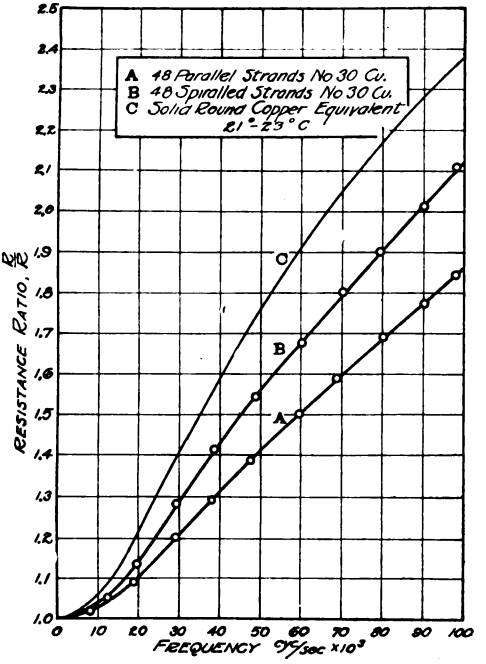

FIGURE 8—Effects of Stranding with Insulated Strands

Measurements with Definitely Spaced Strands.—In order to study, in greater detail, the effect of separating the strands of an unspiralled stranded conductor, a test loop was constructed, about 10 meters long, of a copper conductor comprising 7 strands of number 22 A. W. G. wire (diameter 0.644 mm.). These strands were supported parallel to each other in air, at the positions they would occupy in any 7-strand cable; i. e., one wire at the center, and the six others forming a hexagon about this, as indicated in Figure 9. The spacings were, however, changed in the different tests, by means of specially prepared insulating frames, strung on the wires at suitable distances. In this way, the interaxial spacing distance could be varied from

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1 strand diameter, to nearly 10 strand diameters, in 6 successive steps.

The results of the skin-effect resistance-ratio measurements

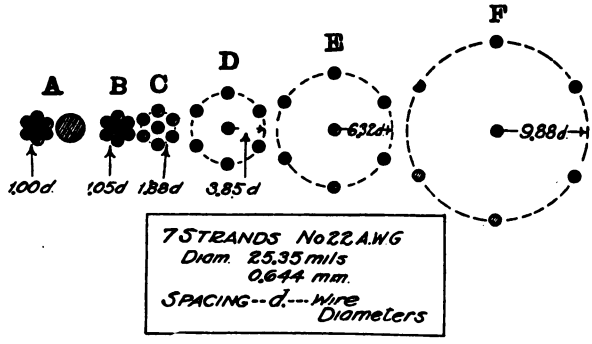

FIGURE 9—Wire Spacing

in these successive geometrical arrangements are presented in the six curves of Figure 10. Curve *A* is for the 7 wires bound

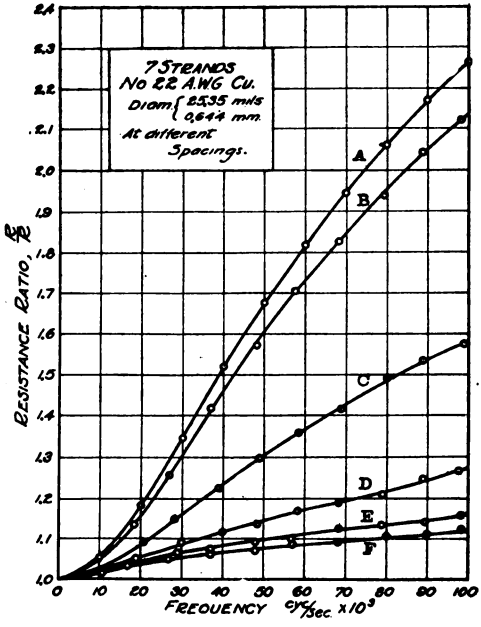

FIGURE 10—Effects of Strand Spacing on the Skin Effect of a Stranded Conductor

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together in close parallel contact. It agrees with the computed resistance ratios of the equisectional solid conductor, within the limits of observational error. Curve B gives the corresponding ratios when all of the 7 wires were insulated with an enamel thickness of approximately 0.015 millimeter, thus increasing the interaxial spacing distance to 1.05 times the strand diameter. It will be seen that the ratio R'/R has thus been lowered about 10 per cent., with respect to number 1, or the equisectional solid wire. The other curves, which will be self explanatory, show how rapidly the spacing distance diminishes the skin-effect resistance ratio. At 100,000 $\sim$, this ratio is only 1.12, for a spacing distance of 9.88 diameters, as against 2.26 for either the solid wire, or a spacing distance of 1.00 diameter. At great separating distances, it appears that the resistance ratio of the 7-strand conductor would substantially coincide with that of any one of its components.

First-approximation formulas have been worked out (see Appendix IV) for dealing with the geometrical relations of such variable "strands," as are indicated in Figure 9. It thus far appears that these formulas represent the observed facts fairly well for the larger spacings, but give lower ratios than are measured at the smaller spacings; perhaps because there is extra skin effect in the component strands at close spacings, due to proximity effect, or current-density distortion due to the magnetic fields of neighboring strands.

The approximate proportionality maintained between the curves in Figure 10 permits of using suitable approximate reduction factors for all frequencies between 40,000 and 100,000 $\sim$. This relation is presented more clearly in Figures 11 and 12. The upper curves of Figure 11 here approach simple hyperbolas; so that doubling the spacing distance approximately halves the extra resistance of skin effect.

Very Finely Stranded Wire.—Two 10-meter samples of highly subdivided wire, with enamel-insulated strands, were obtained thru the courtesy of Dr. A. N. Goldsmith of New York. The first (Figure 13) contained 169 wires, arranged in 13 twisted bundles, each of 13 twisted strands, of number 40 A. W. G. diameter 0.080 millimeter, enamelled to about 0.085 millimeter. The total diameter over all wires, and under the braiding, was approximately 1.47 millimeters, representing a diameter enlargement ratio of about 1.4 with respect to the equisectional solid copper wire. It is obviously difficult to remove the enamel successfully from all of these little wires, and to solder them to-

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gether into a common terminal with the certainty that none have failed to make good electrical contact therewith. Moreover, it would be very difficult to make sure that each of the individual insulated strands is intact, without discontinuity, thruout the test piece. In fact, the linear resistance of the

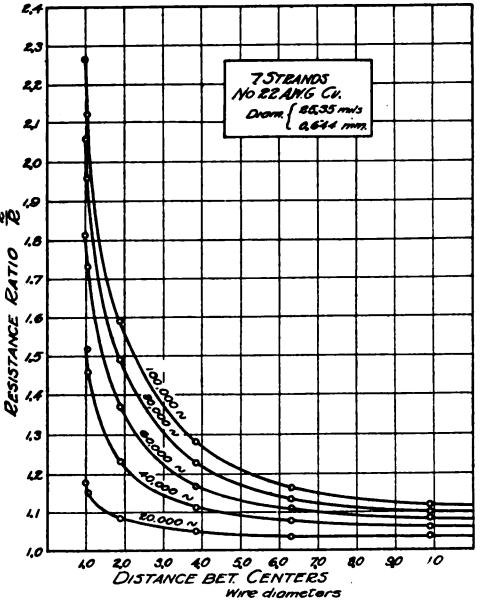

FIGURE 11—Curves Showing Drop in Skin Effect with Increased Spacing Distance between Strands of Conductor

sample was observed to be 0.0221 ohm per meter at 20° C.; whereas, without allowance for spirals, the computed linear resistance was 0.0204 ohm per meter, if all the wires were active, indicating a conductance defect from all causes of 8.4 per cent. However, treating the sample as a stranded wire of, say, 162 strands, having this observed linear resistance, the measured skin-effect resistance-ratio curve is given in Figure 13. It will be observed that, at 100,000 ~, the ratio R'/R is 1.17, and that the extra resistance of skin effect is roughly 60 per cent. less than that computed for the round solid copper wire of equal d. c. linear resistance. The reduction may be attributed partly to the insulation separation of the strands, and partly to transposition effects; since the twisting of the component strands in each

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bundle tends to interchange their relative positions in the cross-section of the whole.

The second sample was made up, in a similar manner, of 49 strands, each number 38 A. W. G. (0.1007 mm. enamelled),

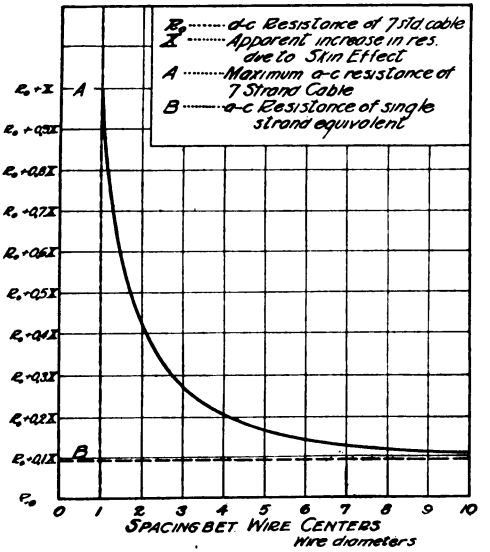

FIGURE 12—Average Decrease in Skin Effect of 7-Strand Cable with Increased Wire Spacing

arranged in 7 bundles of 7 strands each. The observed linear resistance indicated that all of the 49 strands were probably intact and connected in parallel between the soldered extremities. The total cross-section of conductor is approximately 0.39 square millimeter. The unbraided diameter was 0.83 millimeter, representing a diameter enlargement ratio of about 1.5. The skin effect of this strand at various frequencies is shown in Figure 14, and is similar in regard to equivalent round wire to that shown in Figure 13 for the other and more finely subdivided stranded wire.

Litzendraht Wire.—Thru the courtesy of Dr. Goldsmith, a sample of "Litzendraht" wire, 13 millimeters long, was secured for test. This wire is constructed by braiding enamelled copper wires, stocking fashion, in the form of a continuous long cylinder. This sample contained 48 number 38 copper strands, in 16 groups of 3 each. Each strand had a diameter of approximately 0.1

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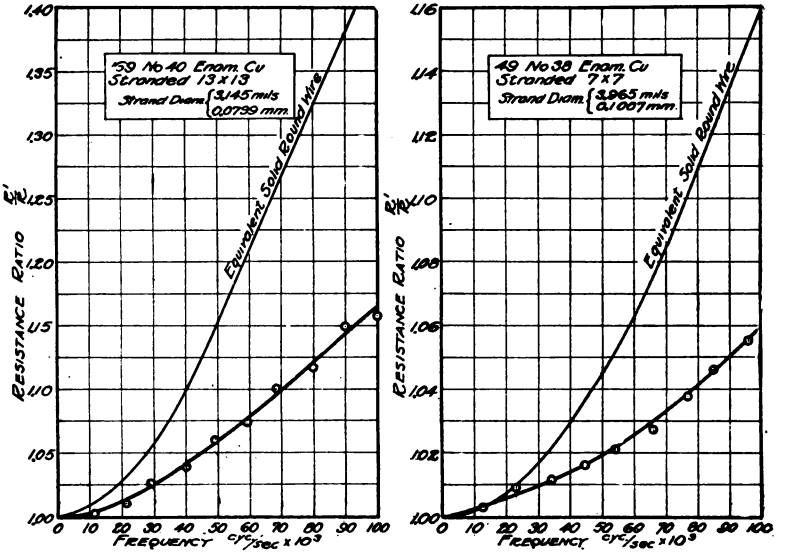
millimeter, enamelled to about 0.12 millimeter. Such a basket braiding necessarily provides a certain central air space, of diameter depending on the braiding and tension. The external diameter, under a heavy braiding of silk, was approximately 1.15 millimeter (0.045 in.), representing a diameter enlargement ratio of nearly 1.7. Such a construction involves a trans-



FIGURES 13 and 14—Skin-Effect Resistance Ratios of Finely Divided and Interwoven Wires

position of strands at regular intervals, between the inner and outer layers, also a hollow cylindrical cross-section. The results of the test of this sample are given in Figure 15. The total cross-section of conductor is approximately 0.38 square millimeter, or nearly the same as in the previous case, but the skin-effect resistance ratio is only 1.016 at 100,000 ∞.

Spirality Effect in Stranded Wires.—Thru the courtesy of the Simplex Wire and Cable Co., six samples of stranded bare copper wires were obtained, and of the same particulars and construction, except in regard to the degree of spiralling or lay. It was considered desirable to ascertain, with the aid of these conductors, the influence of spiralling upon their skin-effect resistance ratio. It had already been ascertained that the resistance of a 7-strand conductor, with the strands uninsulated,

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and in contact without spiralling, is the same as that of the solid equisectional round wire of the same metal and temperature. Consequently, any increase in the resistance ratio of 7-strand uninsulated and spiralled conductors could be attributed to spirality effect, assuming that the extra length of the external strands in the spiralled conductors was taken into account.

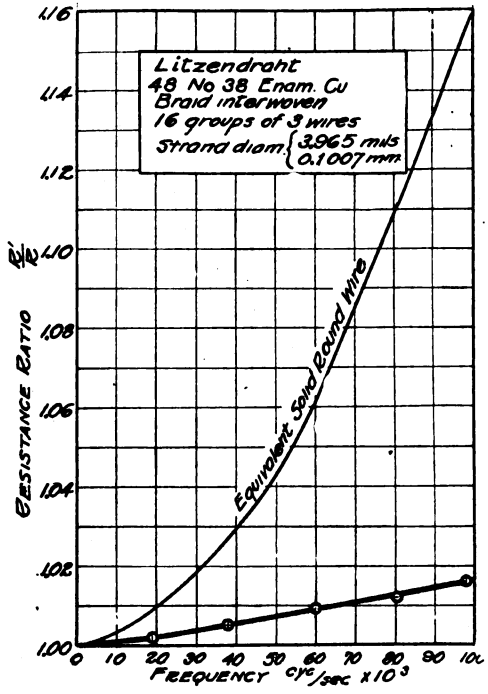

FIGURE 15—Skin-Effect Resistance Ratio for Litzendraht

Each of the six special test conductors was 25 meters long, and consisted of seven strands of bare tinned copper wires, each number 22 A. W. G., of diameter 0.668 millimeter (0.0263 in.), made up in the usual manner by a stranding machine, except in regard to the lay. The lay varied between 1 turn in 7.62 centimeters, to 1 turn in 1.52 centimeter; i. e., a spirality of 0.131 to 0.656 turns per centimeter. To assist in maintaining insulation between these bare copper strands, they were passed thru oil before entering the stranding machine in each case. They were then braided and thus held closely in contact.

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As might be expected, it was found that the skin-effect resistance ratio, at any one frequency, such as 100,000 ∞, increased with the spirality. Figure 16 shows the results obtained with the conductor of greatest spirality, (0.656 turn per centimeter). At 100,000 ∞, the observed resistance ratio was 2.51; whereas the ratio computed for an unspiralled conductor of the

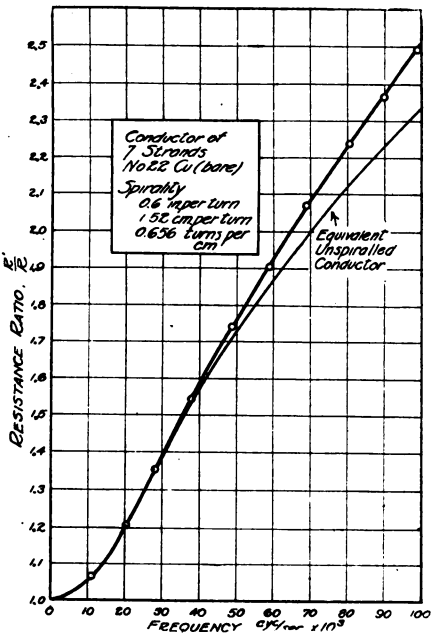

FIGURE 16—Increase in Resistance Ratio of 7-Strand Conductor Due to Spirality

same linear resistance was 2.34. This is the maximum deviation observed among the six samples tested. The corresponding curves for the other samples are omitted, partly because they would lie very close together, and partly because they are not strictly comparable on the same curve sheet, owing to small differences in d. c. linear resistance. In the case of the sample with least spirality, the resistance ratios were 2.41 observed, and 2.38 computed for the solid wire.

The extra linear resistance of spirality may perhaps, as already mentioned, be attributed to loss of power in the substance of the conductor accompanying the longitudinal magnetic field within the spiral. It may therefore be regarded as an extra

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percentage of the d. c. linear resistance at normal temperature. On this basis, Figures 17 and 18 have been drawn. Figure 17 shows the extra percentage resistance of spirality in four different samples plotted against different frequencies; while Figure 18 shows the same percentage resistance at six different frequencies, plotted against spirality. The Figures magnify the spirality

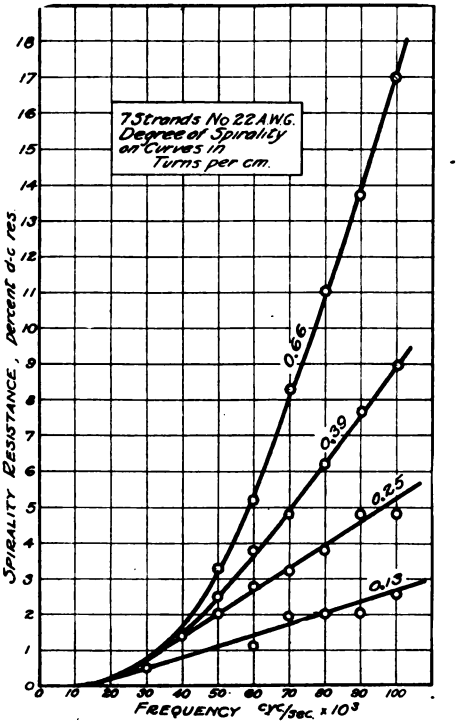

FIGURE 17—Loss Due to Strand Spiralling for Different Degrees of Spirality

effect at the expense of some apparent loss in precision. It will be seen that the spirality effect is almost negligible up to 30,000 ∿. Above 50,000 ∿, the spirality extra resistance is nearly proportional to the frequency. Moreover, the spirality resistance is roughly proportional to the spirality, especially in the neighborhood of 70,000 ∿.

Summing up the results for stranded wires, it appears that the mere subdivision of a solid round wire into seven uninsulated parallel round strands, has no appreciable effect upon the skin-

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effect resistance ratio. The separation of the strands, as by their insulation, rapidly diminishes the resistance ratio, but increases the reactance ratio of the conductor. Spiralling the strands, to the extent ordinarily employed, slightly increases the resistance ratio.

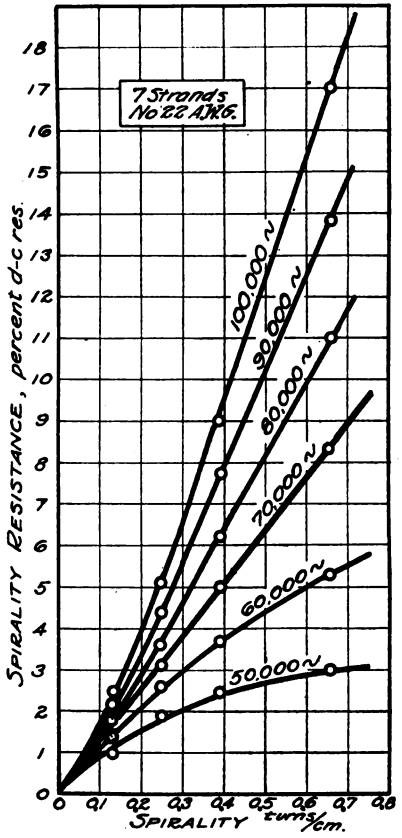

FIGURE 18—Variation in Loss with Degree of Spiralling

Copper Strips—Several samples of copper strip 0.003 inch (0.0076 cm.) were supplied by the courtesy of the Western Electric Co. These were tested in various widths from 0.356 centimeter to 3.8 centimeters (0.14 inch to 1.5 inch). The observed skin-effect resistance ratios are indicated in Figure 19, for five different widths of strip, the abscissas being impressed frequencies, and the ordinates R'/R. Each tested strip was

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mounted in a horizontal loop, or loops, on the testing-room wall. The length of each tested loop varied from about 3 meters, for the narrowest strip, to 25 meters for the widest strip. The spacing between the sides of the loop or loops, varied from 10 to 20 centimeters, and the proximity effect was found to be negligible under these conditions. It will be observed that at

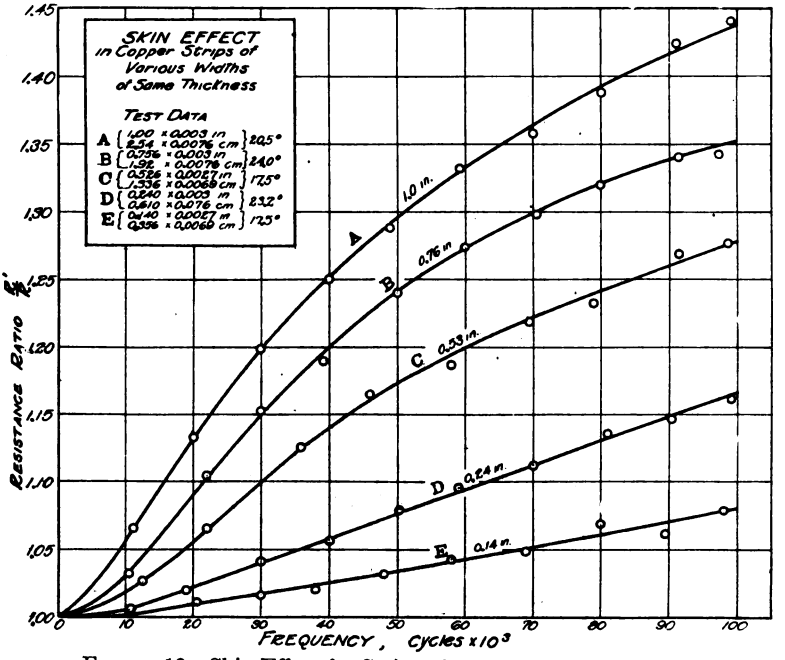

FIGURE 19—Skin Effect for Strips of Different Width but the same Thickness

100,000 ∞, the resistance ratio varies from 1.08 for the 0.356-centimeter strip width, to 1.44 for the 3.8-centimeter width. Owing to slight differences of thickness and of test temperatures the curves are not precisely comparable; but they resemble the curves of resistance ratio for wires of increasing diameter, and the extra resistance is, to a first approximation, proportional to the width of the strip. This relation is presented more clearly in the series of curves of Figure 20, which are taken from the last Figure, for selected frequencies. The abscissas are strip widths, in inches and cm., and the ordinates are resistance ratios. The curves are seen to be, within the limits of frequency presented, rough approximations to straight lines, particularly in the neigh-

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borhood of 30,000 ∿. It is evident that for 0.76-millimeter strip, the effect of increasing width is to increase the resistance ratio in somewhat the same manner as increasing diameter in a round wire, so that a 4-centimeter strip at 100,000 ∿, is by no means twice as good a conductor as a 2-centimeter strip. A strip at radio frequencies is, however, always a better conductor than

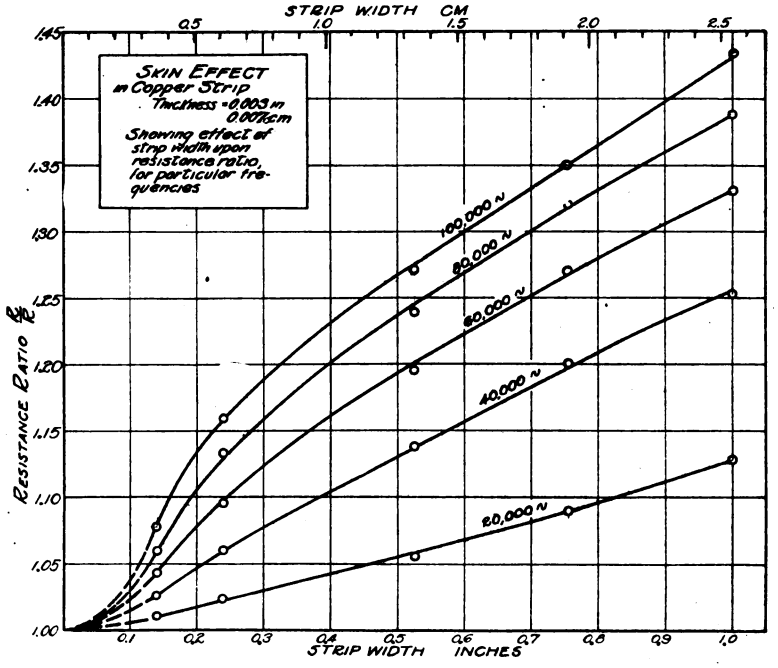

FIGURE 20—Skin Effect Variations with Strip Width, for Particular Frequencies

an equisectional round wire of the same material. Thus, Figure 21 shows the resistance ratio of the 3.8-centimeter strip, by comparison with that of an equisectional round wire 0.192 centimeter in diameter. Up to the frequency of 16,000 ∿, the round wire is the better conductor; but for all higher frequencies, the strip shows distinctly increasing advantages.

Similar tests were made upon samples of strip 0.16 millimeter thick (0.0063 in.) and of widths from 0.64 centimeter to 3.81 centimeters. The comparison between these strips and the solid round copper wire, equisectional to the widest, is shown in Figure 22. It will be observed that the curves are of the same general

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appearance as those for the 0.76 millimeter strips, yet they rise somewhat higher, owing to greater skin-effect in thickness.

Formulas for Skin-Effect in Strips—It has been already pointed out* that the Rayleigh skin-effect formula for indefinitely wide strip is quite inapplicable to single strips of ordinary widths. For particular ranges of dimensions and frequency,

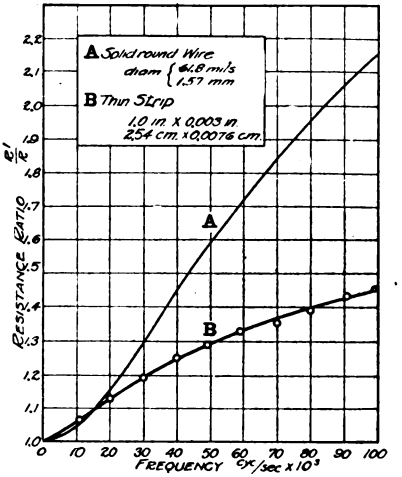

FIGURE 21—Comparative Skin Effects of Thin Copper Strip and Equisectional Round Copper Wire

empirical formulas have recently been suggested.** It is evident experimentally that a long flat strip, carrying an alternating current, sets up a more or less cylindrical magnetic flux distribution about the center, which flux intersects the strip itself. This must give rise to eddy-current e. m. fs. in the strip. If these e. m. fs. produce local eddy-currents in the strip, they would tend to increase the power loss and the effective a. c. resistance. If, however, they produce along the entire length of the strip, an organized e. m. f. distribution, the effect will be to crowd the current toward the edges of the strip, and away from the center; thus producing what may be called an *edge-effect*, perpendicular to that plane.

In order to ascertain which eddy-current action predominated, the local eddy-current effect or the edge-effect, a sample of copper

* Bibliography, Kennelly, Laws & Pierce, number 16.
** Bibliography, Dwight, number 17.

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strip 2.22 centimeters wide (7/8 inch) and 13 meters long, was slit longitudinally near the edges with a sharp knife, in the manner indicated in Figure 23. These slits were from 6 to 8 centimeters

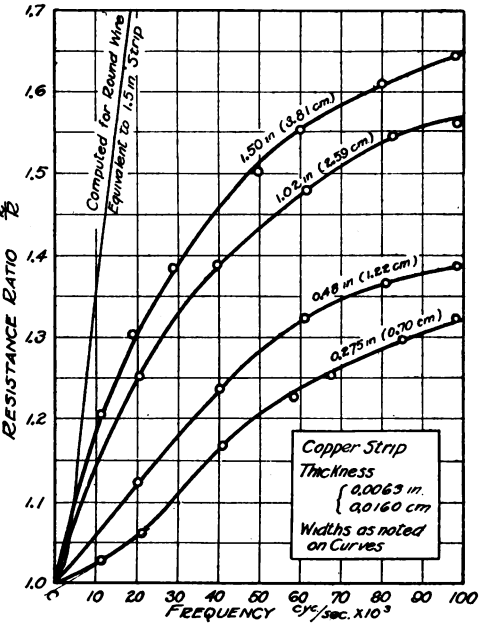

FIGURE 22—Skin-Effect for Strips of Different Width but the Same Thickness.

long, and then ceased for a distance of about 0.5 centimeter, the gaps in the slits overlapping one another; so as to maintain the structural rigidity of the conductor, while at the same time greatly hampering the circulation of eddy-currents across the strip. A strip of similar dimensions was also mounted without any slits. Up to 100,000 $\sim$, no appreciable difference could be detected in the skin-effect resistance ratios of the slitted and unslitted strips. The inference may, therefore, be drawn that the large increase in skin-effect resistance ratio, observed in copper strips, with reference to what should be found in infinitely wide strip, is attributable, at least in large measure, to edge-effect.

We may, therefore, consider, for purposes of discussion, that any long straight conductor of rectangular cross-section, such as a bar, prism, strap or strip, has a skin-effect in two directions,

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namely across its thickness and along its width. The former is commonly regarded as a skin-effect, and the latter may be called, in contradistinction thereto, an edge-effect; but actually, the two effects are essentially manifestations of one and the same phenomenon; that is, a tendency of alternating current to leave the center, and to crowd into outlying positions of the cross-

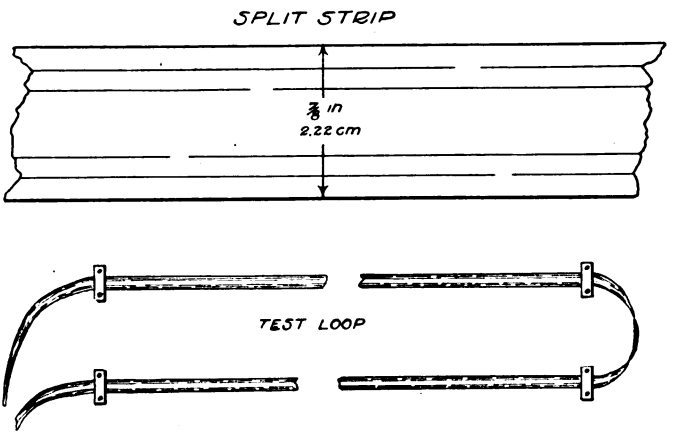

FIGURE 23—Strip Split to Reduce Eddy Currents

section. This distortion of current density may be regarded in either of two ways; namely (1) as due to imperfect penetration of the current from the outlying positions towards the center, (2) as due to the opposing mutual inductive influence, (see Appendix IV) of parallel filaments, whereby the retarding and diminishing effects of mutual inductance are greatest at the center, and least at most remote points. According to either of these ways of viewing the matter, it must appear that the tubular form of conductor can be made to offer the minimum skin-effect at any given frequency, by reducing the wall thickness to a minimum. In the case of a rectangular section, however, the advantage of thinness may be overbalanced by the disadvantage of width, the edge-effect increasing as the skin-effect is reduced. There will thus be a best form of cross-section for each particular cross-sectional area, and frequency.

Parallel Opposed Strips in Close Proximity—In view of the interesting fact that in the case of a pair of parallel going and returning flat strip conductors, in close juxtaposition, one above the other, and separated only by a strip of thin insulator, the

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skin-effect resistance ratio at frequencies below 5,000 ∿ is much lower than when the strips are widely separated, it seemed important to check this result at frequencies up to 100,000 ∿. It has been pointed out by Mr. H. B. Dwight, that the data for such surface-juxtaposed parallel strips, published in the A. I. E. E. paper of 1915,* follow a certain formula which he developed, and which, as pointed out in Appendix IV, coincides with the formula for infinite strips given in that paper, when X is taken as the full thickness, instead of the half thickness of the strip, as is necessary for infinite widths:

$$\frac{Z}{R} = \frac{a X}{tanh\,(a X)} \qquad \text{numeric} \angle \qquad (2)$$

This case constitutes the first reliable skin-effect formula for flat strips of finite width known to the writers.† Available formulas for finite strips seem to be all based on Rayleigh's infinite-strip formulas, and are therefore open to very large correction for edge-effect.

The test was made with a loop 6.1 meters long, of two flat strips, each 2.54×0.0076 centimeters (1.0×0.003 inch) laid on a flat board, the lower strip being entirely enveloped in tissue paper, 0.0038 centimeter (0.0015 inch) thick, and the upper strip laid carefully over this, so as to keep the edges parallel. Little bindings of paper were then applied to the two insulated strips, after which weights were applied uniformly to the system. Great care had to be taken to secure good alignment of the edges in the opposing strips, since it was found, in preliminary trials, that a relatively small amount of overprojecting at the edges, added materially to the losses and apparent resistance of the loop.

The d. c. resistance of the loop at 20° C. was 0.102 ohm, and the a. c. resistance at 93,000 ∿, was the same, within the limits of observational error; so that the skin-effect resistance ratio up to this frequency was sensibly 1.00. In such a disposition of conductors, virtually forming a paper condenser, the susceptance of the dielectric may have an appreciable effect on the measurement. In this case, however, the loop was so short that the error due to capacity susceptance was negligible.

Copper tubes are of especial interest in relation to skin-effect; since, as already mentioned, their resistance ratio should be capable of being made lower than that of any other form of

* Bibliography, number 16.

† A somewhat similar case is, however, discussed by Lord Rayleigh, Bibliography, number 1.

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conductor. A tube of large diameter and very thin wall should presumably have a skin-effect resistance-ratio calculable on the basis of infinitely wide strip with X, see formula (2), taken as equal to the wall thickness.

Two copper tubes were tested. One was a slit tube of 2.22 centimeter (7/8 inch) perimeter, made by rolling a length of the thin flat strip previously tested (2.22 cm. wide $\times$ 0.076 mm. thick) over a cylindrical wooden form. This produced a slit copper tube 12 meters long, and of an external diameter approximately 7.2 millimeters. The calculated skin-effect resistance ratio of 0.076 millimeter copper strip of indefinitely great width is, by (2) 1.005 at 100,000 $\sim$. When the tube conductor made of this strip was tested at 100,000 $\sim$, its skin-effect resistance ratio was found to agree with this value, within the limits of observational error. The same strip, when tested flat, before being rolled into tube, was found to give 1.38 at 100,000 $\sim$. This experiment gave a striking demonstration of the influence of edge-effect in one and the same single loop of copper conductor. With the edge-effect present in the flat strip, the a. c. resistance at 100,000 $\sim$ was nearly 40 per cent. greater than the d. c. resistance. With the edge-effect removed, by bending the same strip into a simple slit tube, the a. c. resistance, at the same frequency, was hardly greater than the d. c. resistance.

The second tube was a soft drawn continuous copper tube of external diameter 3.18 millimeters and of wall thickness 0.65 millimeter (number 22 A. W. G.). Its length was approximately 25 meters, in two horizontal loops, or four horizontal passages, along the wall, with an interaxial distance of about 20 centimeters between adjacent conductors.

Figure 24 shows the observed skin-effect resistance ratios of this tube at various frequencies up to 100,000 $\sim$. It will be observed, that at 100,000 $\sim$, this ratio is 2.46. The round wire of equivalent cross-section, shown in dotted lines, would have, at the same frequency, a ratio of 3.17. The curve C shows the computed ratios for infinitely wide strip, of the same thickness (0.65 mm.), that is, a tube of infinite diameter and this wall thickness; while curve D shows the corresponding ratios for a tube of zero internal diameter; that is, a solid wire of radius 0.65 millimeter. It will be seen that the curve OA of observations lies nearly midway between the curve of infinite internal diameter, and of zero internal diameter, having the same wall thickness, on the assumption that the magnetic flux density vanishes within a uniform straight hollow cylinder carrying a

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current. The curve OA corresponds very closely to formula (2) when X is taken as 82 per cent. of the actual wall thickness.

Comparisons of Skin-Effect Resistance Ratios for Different Types of Conductor—When an alternating current of radio-frequency has to be carried with the least loss for a given amount of copper, it is evident that the skin-effect resistance ratio must be made as near unity as possible. It is therefore important to consider what form a cross-section of given number of square

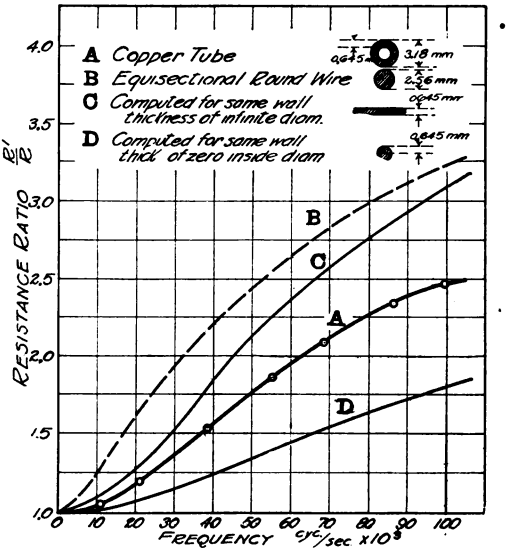

FIGURE 24—Skin-Effect in Copper Tube

millimeters of copper should have, in order to make the conductor most efficient. The best form of the conductor will depend to some extent upon the frequency.

For the particular frequency of 100,000 ~, Figure 25 shows the skin-effect resistance ratios of various kinds of geometrical section. The ordinates are in R'/R. The abscissas are in square millimeters, and also in square mils (10^{-6} sq. inch) of copper cross-section. It will be seen that round solid wire has the greatest resistance ratio thruout almost the entire range of cross-section shown, altho spiralled 7-strand conductor, devoid of internal insulation, would have a resistance ratio in excess of this by an amount depending on the degree of spirality. All other forms of conductor, for a given cross-section, have less resistance ratio.

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The few tests thus far made of stranded conductors are plotted on the diagram. The diminution in resistance ratio depends upon the number of strands, their internal transposition and separation. Below most of these come three-mil (0.076 mm.) copper strip, which is not so low as six-mil (0.15 mm.) strip. As already explained, the edge-effect of 3-mil strip, which

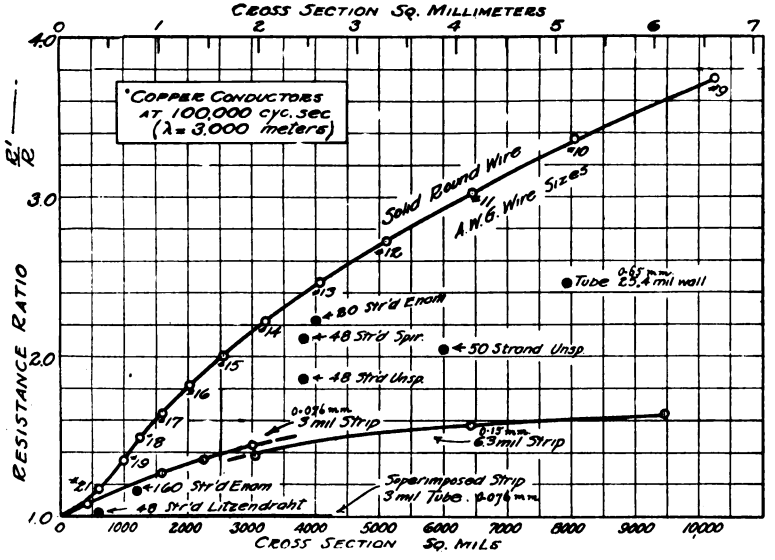

FIGURE 25—Comparative Skin-Effects for Different Types of Conductors

must be twice as wide as 6-mil strip for the same cross-section, predominates over skin-effect at 100,000 ∿, so that there is a disadvantage, within the limits of cross-section shown, in rolling 0.15-millimeter strip into 0.075-millimeter strip for 100,000 ∿. The limit of thickness to which flat strip should be rolled, for a given cross-section, is evidently a function of the frequency, which determines the equivalent penetration skin-depth δ. The tests have not yet been carried sufficiently far to assign the economical thickness limit; except that, at 100,000 ∿, it is above 3 mils (0.076 mm.) for cross-sections less than 6 square millimeters.

The only apparent means of avoiding the edge-effect, so as to permit of using very thin sheets of conductor, is either to use two opposing parallel strips in close proximity, as described in

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Appendix III; or to roll a single strip up into circular tube form. In either case, as Figure 25 shows, the resistance skin-effect ratio of 0.075-millimeter strip fell to almost 1.0. The use of opposing juxtaposed parallel strips, as going and returning conductors, has limitations, owing to the large linear capacitance involved. No matter how high the frequency, there seems to be no electrical limit to the reduction of extra skin-effect resistance, by using tubular conductors of sufficiently thin wall. Mechanical considerations suggest that this might best be accomplished by using a flexible insulating core, with fine copper wires braided over the surface. In such a flexible conductor, by taking thin enough wire, and avoiding spirality effects, the skin-effect should be reduced indefinitely, since there could be no edge-effect.

The data contained in Figure 25 are presented in a somewhat different form in Figure 26, which shows the total linear resistance of copper conductors of different types of cross-section at 100,000 ∿ and 20° C, for different cross-sectional areas as abscissas. By reference to this diagram, the most suitable conductor can be chosen for this particular frequency, so far as the experimental data, thus far obtained, will permit.

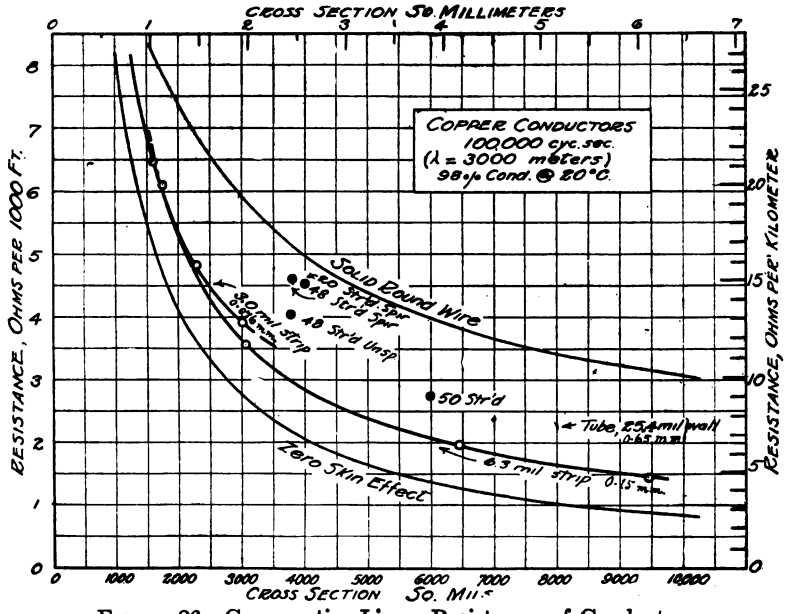

FIGURE 26—Comparative Linear Resistances of Conductors of Different Types at 100,000 Cycles

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SUMMARY

The following conclusions have been reached from tests at frequencies up to 100,000 ~, on straight conductors, shielded from proximity effects, and of cross-sections up to 6 square millimeters.

(1) The skin-effect resistance ratio of round copper wires has been found to conform to the standard Heaviside-Kelvin Bessel-function formulas, within the limits of experimental error.

(2) The skin-effect resistance-ratio of a copper conductor formed of seven equal and parallel round bare strands, (six surrounding a central seventh), was found to be the same as that of the equisectional solid round wire up to 100,000 ~, within the limits of observational error. It is therefore inferred that the subdivision of a round wire into a round cable of uninsulated contacting strands does not alter the skin-effect.

(3) The effect of simple spiralling of strands in the same direction, increased the resistance ratio.

(4) The resistance ratio of a subdivided wire, with parallel insulated strands, fell below that of the equisectional solid wire, and diminished rapidly with the spacing, or degree of strand separation.

(5) The braiding of strands, so as to effect their transposition in the cross-section at frequent intervals, was found to diminish the skin-effect.

(6) The skin-effect in copper strips was found to be usually, but not invariably, less than that of equisectional round wires.

(7) Increasing the width of a copper strip within the limits reported, was found to increase the skin-effect resistance ratio; owing to what may be called "edge-effect."

(8) A pair of parallel going and returning flat strips, separated by a thin insulating layer, was found to have a skin-effect depending only on the strip thickness; that is, without perceptible edge-effect.

(9) Rolling a flat strip into the form of a slit tube destroyed the edge-effect, and reduced the resistance ratio to a minimum, dependent only on the strip thickness.

(10) Cutting longitudinal slits with a sharp knife in a thin flat copper strip, was found not appreciably to affect its skin-effect resistance ratio.

(11) In order to employ a stranded wire of minimum skin-effect at radio-frequency, it seems desirable to employ thick insulation on the strands, such as double cotton, to transpose

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the strands by braiding, and to avoid spirality in one and the same direction.

(12) In order to employ a flat strip conductor most effectively, it is necessary to stop rolling it out laterally when the increasing edge-effect more than offsets the reduction in skin-effect.

(13) The hollow tubular form seems to be the most efficient type of radio-frequency metallic conductor, since, with proper mechanical internal support, the skin-effect can be indefinitely diminished by diminishing the wall thickness, and the edge-effect is absent.

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APPENDIX I

DETAILS OF APPARATUS AND PROCEDURE FOR MEASURING SKIN-EFFECT RESISTANCE RATIOS, AT FREQUENCIES BETWEEN 10,000 ∞ AND 100,000 ∞

The detailed electrical connections of the induction bridge referred to above, under the title "Inductive Bridge Method," are presented in Figure 27. The alternator *A* supplies a current of from 4 to 5 amperes r. m. s. thru the hot-wire ammeter *M* to the bridge, with the aid of the adjustable condenser C_3 of

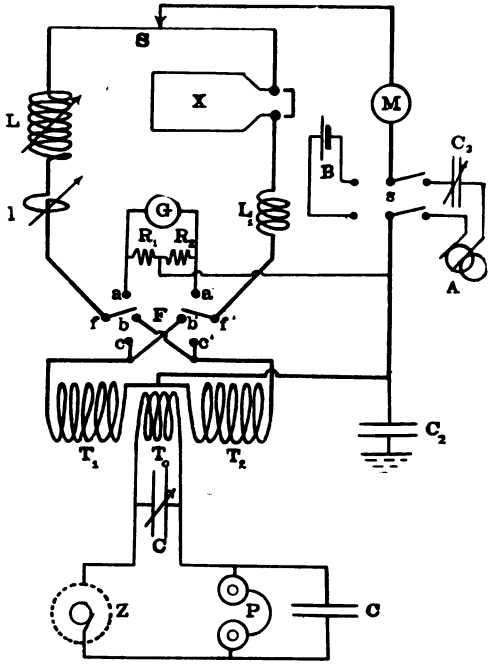

FIGURE 27—Detailed Diagram of Differential Bridge Circuit as Employed in Tests.

about 1 μf. (microfarad), which partially compensates the inductive reactance of the bridge circuit. The adjustable bridge resistance is a single slide-wire meter of Ia-Ia high-resistivity wire, number 18 A. W. G. (diameter 1.02 mm.) offering 0.0058 ohm per linear centimeter, or 0.58 ohm in all. The resistance measurements are all taken in terms of lengths of this slide wire, which possesses negligible skin-effect.

The test conductor *X* is connected between two stout ter-

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minals containing mercury, that can be shorted by a heavy copper link. A bridge balance is obtained by adjusting L and S, first with X shorted, and then with X unshorted. The difference between the readings of S in these two tests, measures the resistance of X at the impressed frequency.

The switch s changes from the test dry battery B, of about 2 volts, for d. c. measurements, to the alternator A, for a. c. measurements. The switches ff' are thrown to aa', for d. c. measurements, and to bb' for a. c. measurements. The contacts cc' provide for the reversal of bb', that is, for the exchange of the two similar and equal inductive arms T_1 and T_2. Each of these is wound in two layers of 8 turns, or 16 turns all told, in a groove 1.5 centimeter wide cut in a wooden cylinder, the mean winding diameter being 15 centimeters. The wire in T_1 and T_2 consists of 9 strands each of 9 enamel-insulated number 30 A. W. G. (0.25 mm.) wires, or 81 insulated number 30 wires in all. The mid-point between these two coils T_1 T_2 is connected to ground thru an adjustable condenser C_2 of from 0.5 to 4 μf., which helps to minimize electrostatic disturbances in the detector P. The secondary winding T_0 consists of 180 turns of plain double cotton-covered number 18 A. W. G. copper wire (1.02 mm. diam.) equally distributed in two grooves in the wooden cylinder, one on each side of the primary winding.

The fixed inductance L_1 has 20 turns in 2 layers of 90 insulated strands number 30 A. W. G. wire. The mean winding diameter is 8 centimeters and winding breadth 3.8 centimeters. The object of this inductor is to assist the test wire X to make up an inductance within the range of the inductance variometer L. This variometer has an outer fixed coil and an inner frictionally movable coil, the angle between the axes of the two being adjustable by hand. Each of the two coils has 12 turns in 2 layers. The mean diameter of the pair is 15 centimeters. The wire has 90 enamelled strands of number 30 A. W. G., and the total maximum inductance is about 0.1 mh. After an approximate inductance balance has been reached in L, a yet finer adjustment to balance can be made with the single turn l, suspended beneath the testing table, and capable of being altered in its area, and therefore in its self-inductance, by a winding spindle controlled from a distance, so that the observer's hand does not have to reach over the table when making the final adjustment.

The tuning adjustable condenser C has about 4 m.μf. (millimicrofarads) and C_1, the condenser shunting the detector tele-

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phones, about 3 m. μf. C is adjusted for best hearing with the impressed frequency. The frequency being ordinarily above the limit of audibility, the commutator interruptor Z, motor-driven at approximately 1,000 makes and breaks per second, produces an audible effect in the head telephones P, which have each a d. c. resistance of approximately 1,000 ohms.

For the d. c. measurements, the equal-resistance arms R_1 R_2 have each 0.25 ohm, and the portable galvanometer G has 250 ohms, with a sensitivity of 1 scale division for 1 microampere.

To make a test of a conductor X, it is connected to the shorted binding posts. The switch s is thrown to the left, f to a and f' to a'. A d. c. Wheatstone-bridge balance is then obtained on G, by adjusting the position of the slider S. The short is then removed from X, and the balance repeated, at a new position of S. The change in the slide-wire reading measures R, the d. c. resistance of X. Arrangements, not shown in Figure 27, also permit of reversing the arms R_1 and R_2, so as to reveal any inequality they may possess.

The switch s is then thrown to the right, the alternator A having first been brought to the desired speed and frequency. Switch f is thrown to b and f' to b'. The alternating current in the bridge circuit is observed at M, and adjusted to between 4 and 5 amperes r. m. s., with the aid of the condenser C_3. The conductor X being shorted, a balance for both resistance and inductance is obtained telephonically at P, by successive adjustments of S and L. It is important that the observer should take up a stationary position with respect to the apparatus, and that the final adjustments should be made at S and l, without changing his position. The short is now removed from the conductor X, and a new balance obtained at S and $L + l$. The change in the reading of S measures R', the a. c. resistance of X, including the skin-effect. Reversals of f and f' enable average values to be secured.

With careful manipulation, the skin-effect resistance ratio in specimens of wire X, whose total d. c. resistance lies preferably between 0.2 and 0.4 ohm, may be measured to within a deviation probable error, for a single observation, of about $\frac{1}{3}$ of 1 per cent.

A photograph of the testing table is shown in Figure 28, with the elements marked to correspond with Figure 27.

It has been found advisable to select a pair of head telephones with cases of insulating material instead of metal, since the capacitance disturbance thru the body of the observer on the bridge balance is thereby greatly reduced.

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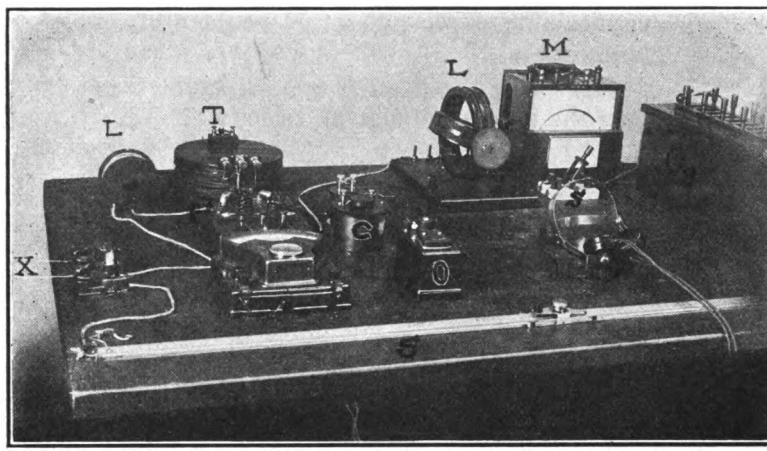

FIGURE 28—Photograph of Testing-Table Apparatus

APPENDIX II

COMPUTATION OF SKIN-EFFECT RESISTANCE RATIO IN ROUND WIRES

The theory of skin-effect in round wires has been developed and published by a number of investigators, commencing with the work of Clerk Maxwell in 1873. A bibliography of the subject appears in the paper by Kennelly, Laws and Pierce in the "Transactions of the American Institute of Electrical Engineers," September, 1915. The formula, as developed in that paper, following Jahnke and Emde is

$$\frac{Z}{R} = \frac{a_o X}{2} \cdot \frac{J_o(a_o X)}{J_1(a_o X)} \qquad \text{numeric} \angle \qquad (3)$$

where Z is the linear impedance of the wire at the impressed single frequency f^{∞}, R is its linear resistance to steady currents. X is the radius of the wire in centimeters, $J_o(a_o X)$ and $J_1(a_o X)$ are respectively Bessel functions of zero and first order, a_o is the semi-imaginary quantity

$$a_o = \sqrt{-j\,4\,\pi\,\gamma\mu\,\omega} = \sqrt{2\,\pi\,\gamma\mu\,\omega} - j\sqrt{2\,\pi\,\gamma\,\mu\,\omega} = a_2 - j\,a_2 \qquad \text{cm.}^{-1} \angle \qquad (4)$$

where $j = \sqrt{-1}$

$\pi = 3.14159 \ldots$

γ = conductivity of wire at actual temperature $\left(\frac{\text{abmhos}}{\text{cm.}}\right)$

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μ = permeability of wire at actual temperature $\left(\frac{\text{gausses}}{\text{gilberts per cm.}}\right)$

$\omega = 2\pi f$ the impressed angular velocity $\left(\frac{\text{radians}}{\text{sec.}}\right)$

This formula gives a vector skin-effect impedance ratio

$$\frac{Z}{R} = M \angle \beta^\circ \qquad \text{numeric} \angle \qquad (5)$$

the real component of which is the skin-effect resistance-ratio

$$\frac{R'}{R} = M \cos \beta \qquad \text{numeric} \qquad (6)$$

or

$$\frac{R'}{R} = \left| \frac{a_o X}{2} \cdot \frac{J_o(a_o X)}{J_1(a_o X)} \right| \cdot \cos \beta \qquad \text{numeric} \qquad (7)$$

In the above mentioned paper, the values of a_o (at $\rho = 1724$ absohm-cm.), $J_o(Z \angle = 45°)$ and $J_1(Z \angle = 45°)$, were tabulated and graphed over a certain range. Altho those tables assist computation, yet since for ordinary purposes the resistance ratio $\frac{R'}{R}$ only is required, a simpler basis of computation is obtained by tabulating and plotting the values of (3) assuming a certain representative value of γ.

Figure 29 gives the graph of $\frac{R'}{R}$ with reference to $\sqrt{\frac{f}{R'_o}}$, and also with reference to $| a_o X |$, as abscissas. It will be noticed that for values of $| a_o X |$ greater than 4.0, the curve approximates to the straight line

$$\frac{R'}{R} = \frac{| a_o X |}{2\sqrt{2}} + \frac{1}{4} = \frac{a_2 X}{2} + \frac{1}{4}.$$

Russell's formula* for the ratio $\frac{R'}{R}$ in a round wire, and using the notation of this paper, can be put in the form

$$\frac{R'}{R} = \frac{a_2 X}{2} + \frac{1}{4} + \frac{3}{32 a_2 X} - \frac{1}{16 (a_2 X)^3} + \cdots \qquad \text{numeric} \qquad (8)$$

It is evident that for large values of $a_2 X$, we may, for practical purposes, ignore all terms after the second, as is shown by the curve in Figure 29. We may also arrive at the same result by considering the depth of the equivalent skin δ centimeters in a plate conductor carrying rapid alternating currents; i. e., such a depth as, without skin effect, or with the full conductivity of continuous currents, would carry the same alternating-current

* Proc. Phys. Soc., London, Jan., 1909.

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strength as the actual plate does in the presence of skin effect. It is shown in the Kennelly-Laws-Pierce paper of 1915 that, for a flat plate, this equivalent skin depth has the value

$$\delta = \left| \frac{\tanh \alpha X}{\alpha X} \right| \frac{X}{\cos \beta} \quad \text{cm.} \quad (9)$$

where X is the half thickness of the plate, α is the semi-imaginary

$$\alpha = \sqrt{j 4\pi\gamma\mu\omega} = \sqrt{2\pi\gamma\mu\omega} + j\sqrt{2\pi\gamma\mu\omega} = a_2 + j a_2 \quad \text{cm.}^{-1} \angle \quad (10)$$

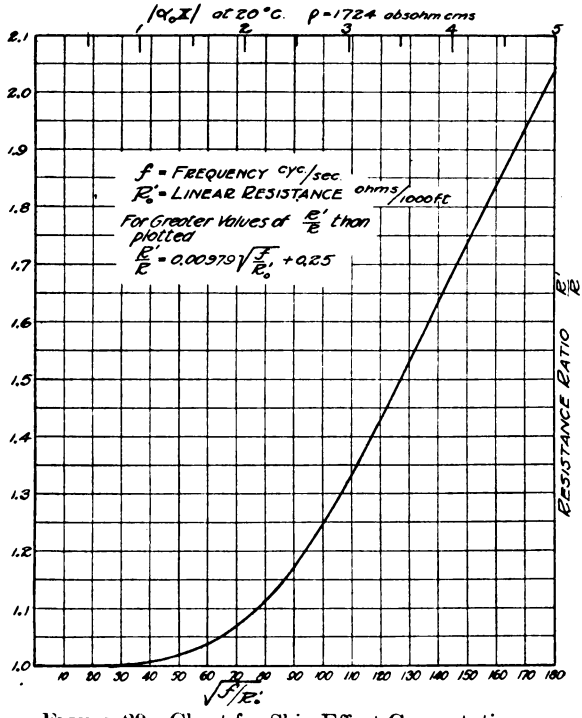
|α₀X| at 20°C. ρ=1724 absohm cms
f = FREQUENCY cyc/sec.
R₀' = LINEAR RESISTANCE ohms/1000 ft.
For Greater Values of E'/E than plotted
E'/E = 0.00979 √(f/R₀') + 0.25
E'/E RESISTANCE RATIO
√(f/R₀')

FIGURE 29—Chart for Skin-Effect Computations on Solid Round Wires

When $| \alpha X |$ exceeds 6.0, this approximates closely to

$$\delta = \frac{1}{a_2} = \frac{1}{\sqrt{2\pi\gamma\mu\omega}} \quad \text{cm.} \quad (11)$$

Applying this plate theory to a round wire of such dimensions that we may ignore the curvature of the surface, the linear resistance of the wire with continuous currents will be

$$R = \frac{\rho}{\pi X^2} \quad \frac{\text{absohms}}{\text{cm.}} \quad (12)$$

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where $\rho = \frac{1}{\gamma}$ is the resistivity. The linear resistance of the same wire to radio-frequency alternating currents will be

$$R' = \frac{\rho}{\delta \cdot 2\pi\left(X - \frac{\delta}{2}\right)} \quad \begin{matrix}\text{absohms} \\ \text{cm.}\end{matrix} \qquad (13)$$

since the current is now supposed to be carried entirely by a superficial cylinder of depth δ centimeter and mean radius $\left(X - \frac{\delta}{2}\right)$ centimeter. The skin-effect resistance ratio of the wire will therefore be

$$\frac{R'}{R} = \frac{X^2}{2\delta\left(X - \frac{\delta}{2}\right)} = \frac{X}{2\delta\left(1 - \frac{\delta}{2X}\right)} \qquad \text{numeric} \quad (14)$$

and since $\delta/(2X)$ is a small quantity with respect to unity, we may take

$$\frac{R'}{R} = \frac{X}{2\delta}\left(1 + \frac{\delta}{2X}\right) = \frac{X}{2\delta} + \frac{1}{4}$$
$$= \frac{a_2 X}{2} + \frac{1}{4} \qquad \text{numeric} \quad (15)$$

which is Russell's formula as far as two terms.*

For practical work with round non-magnetic wires, for which $\mu = 1$, it may be noted that if R_o be the resistance in ohms of a wire of length l centimeters; then for all frequencies

$$\frac{a_2 X}{2} = 3.162 \times 10^{-5}\sqrt{l\pi} \cdot \sqrt{\frac{f}{R_o}} \qquad \text{numeric} \quad (16)$$

for a wire of R_o, expressed in ohms per meter ($l = 100$)

$$\frac{a_2 X}{2} = 3.162 \times 10^{-4}\sqrt{\pi} \cdot \sqrt{\frac{f}{R_o}}$$
$$= 5.6 \times 10^{-4} \cdot \sqrt{\frac{f}{R_o}} \qquad \text{numeric} \quad (17)$$

If, however, we use R'_o as the resistance of 1,000 feet of wire as found in the ordinary wire tables ($l = 3.048 \times 10^4$)

$$\frac{a_2 X}{2} = 3.162 \times 10^{-5}\sqrt{3.048\pi \times 10^4} \cdot \sqrt{\frac{f}{R'_o}}$$
$$= 0.00979\sqrt{\frac{f}{R'_o}} \qquad \text{numeric} \quad (18)$$

*A. Russell, *Phil. Mag.*, April, 1909; *Proc. Phys. Soc. London*, volume 21, 1909.

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and $\alpha_o X = 0.02768\sqrt{\frac{f}{R'_o}} = \frac{1}{36.1}\cdot\sqrt{\frac{f}{R'_o}}$ numeric (18a)

Consequently, in terms of wire-table resistances of a given size of wire at a certain temperature, when $\alpha_2 X$ is greater than 3 say, formula (15) reduces to

$$\frac{R'}{R} = 5.6\times10^{-4}\sqrt{\frac{f}{R_o}} + 0.25 \quad \text{numeric} \quad (19)$$

or

$$= 9.79\times10^{-3}\sqrt{\frac{f}{R'_o}} + 0.25 \quad \text{numeric} \quad (20)$$

We may therefore plot a curve in terms of $\sqrt{\frac{f}{R_o}}$, or $\sqrt{\frac{f}{R'_o}}$, as abscissas, and read off the skin-effect resistance ratios as ordinates. For values of $\sqrt{\frac{f}{R'_o}}$ above say 200, the values of $\frac{R'}{R}$ fall near to the straight line of formula (15); see Figure 29.

APPENDIX III

ELEMENTARY THEORY OF A FLAT LOOP FORMED BY A PAIR OF FLAT PARALLEL COPPER STRIPS, ONE IMMEDIATELY OVER THE OTHER AND SEPARATED BY A UNIFORM THIN INSULATING LAYER

Let the width of each strip be b centimeters and d the thickness in centimeters of the thin insulating layer. The skin-effect resistance and inductance ratios of two such copper-strip loops of different sizes was recorded in Figures 14 and 15 of the paper by Kennelly, Laws and Pierce, in the A. I. E. E. Proceedings for August, 1915. It was discovered by Mr. H. B. Dwight,* that these curves correspond almost exactly (in the symbols here employed) with the formula:

$$\frac{Z}{R} = \frac{1+\frac{(\alpha X)^2}{2!}+\frac{(\alpha X)^4}{4!}+\frac{(\alpha X)^6}{6!}+\cdots}{1+\frac{(\alpha X)^2}{3!}+\frac{(\alpha X)^4}{5!}+\frac{(\alpha X)^6}{7!}+\cdots} \quad \text{numeric}\angle \quad (21)$$

where $\frac{Z}{R}$ is the skin-effect impedance ratio of the loop, X is the thickness in centimeters of each strip, and α is a semi-imaginary quantity (affected by $\angle 45°$) defined below.

But this formula reduces to

$$\frac{Z}{R} = \frac{\alpha X}{\tanh \alpha X} \quad \text{numeric}\angle \quad (22)$$

* Bibliography 20.

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which is the skin-effect impedance ratio of an infinitely wide strip as given in formula (100) of that paper, differing only in form from Rayleigh's formula, except that, whereas for an infinitely wide single strip, X should be taken as the half-strip thickness, when used for Dwight's reduction it should be taken as the whole thickness. In other words, the formula for infinite strip applies to each of a pair of parallel finite strips, when their surfaces are juxtaposed, and X is taken as the full thickness instead of the half thickness of each strip.

Referring to Figure 30, if $A\ B\ C\ D$ and $E\ F\ G\ H$ are the cross-sections of a pair of similar and parallel copper strips forming respectively the going and returning conductors of a long flat

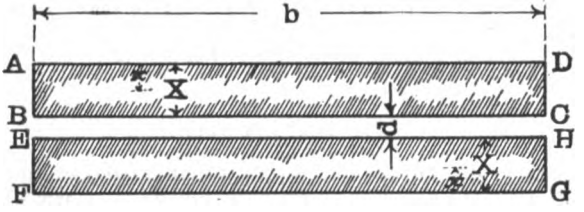

FIGURE 30—Superimposed Strips

loop, the strips being separated by an insulator of permeability unity and uniform thickness d centimeters; the symmetry of the geometrical relations permits of either strip being selected for obtaining the results applying to both. Then the magnetic flux inside this loop will be parallel to the strips except near the edges. Moreover, if $+I$ be the electric current strength (abs-amperes) flowing in the lower strip, and $-I$ that in the upper; then the total magnetomotive force in the magnetic circuit will be $4\pi I$ gilberts. The reluctance of the magnetic circuit will reside almost entirely in the narrow channel of insulator between the strips, and the residual reluctance in the external widely extending return path will be relatively negligible. This means that the $4\pi I$ gilberts will be expended almost wholly in the channel between the strips; where the magnetic field intensity will be very nearly

$$\mathfrak{H} = \frac{4\pi I}{b} \quad \text{gilberts per cm.} \quad (23)$$

producing correspondingly a uniform flux density in the channel

$$\mathfrak{B} = \frac{4\pi I}{b} \quad \text{gausses} \quad (24)$$

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At the outer surfaces of the strips, the flux density will be substantially zero. The differential equation of magnetic instantaneous alternating flux density at any distance x centimeters inwards from the surfaces $F\,G$ or $A\,D$ is then

$$\frac{d^2\mathfrak{B}_x}{d\,x^2} = \alpha^2\,\mathfrak{B}_x \qquad \frac{\text{gausses}}{\text{cm.}^2}\angle \qquad (25)$$

where $\alpha^2 = j\,4\,\pi\,\gamma\,\mu\,\omega$ $\qquad \text{cm.}^2\angle$

$j = \sqrt{-1}$

γ = conductivity of the strip in $\frac{\text{abmhos}}{\text{cm.}}$

μ = permeability of the strip taken as unity

$\omega = 2\,\pi f$ impressed angular velocity $\qquad \frac{\text{gausses}}{\text{gilberts per cm.}}\,\frac{\text{radians}}{\text{sec.}}$

f = impressed frequency $\qquad \frac{\text{cycles}}{\text{sec.}}$

Similarly, the differential equation for current density i_x absamperes per square centimeter at elevation x centimeters is

$$\frac{d^2\,i_x}{d\,x^2} = \alpha^2\,.\,i_x \qquad \frac{\text{absamperes}}{\text{cm.}^2}\angle \qquad (26)$$

The solution of both (25) and (26) is of the same type:

$$i_x = A_1\,cosh\,\alpha x + B_1\,sinh\,\alpha x \qquad \frac{\text{absamperes}}{\text{cm.}^2}\angle \qquad (27)$$

At $x = 0$ or at the outer surface FG of the strip, $cosh\,\alpha\,x = 1$, $sinh\,\alpha\,x = 0$,

$$i_o = A_1 \qquad \frac{\text{absamperes}}{\text{cm.}^2}\angle \qquad (27a)$$

and between (27) and (27a) B_1 can be shown to be zero; so that

$$i_x = i_o\,cosh\,\alpha\,x \qquad \frac{\text{absamperes}}{\text{cm.}^2}\angle \qquad (28)$$

$$\frac{i_{x\,m}}{i_{X\,m}} = \frac{i_{x\,r}}{i_{X\,r}} = \frac{i_x}{i_X} = \frac{cosh\,\alpha x}{cosh\,\alpha X} \qquad \text{numeric}\angle \qquad (29)$$

where the subscripts m and r represent respectively maximum cyclic and r. m. s. values of the current density. If $\alpha\,x$, a semi-imaginary quantity, (of equal real and imaginary components) is defined as the *hyperbolic position angle* corresponding to the layer of distance x from the external surface, the current density varies directly as the hyperbolic cosine of the position angle. Such cosines have all been tabulated and charted. (Bibliography 18.)

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The average r. m. s. current density over the cross-section is

$$i_{qr} = \frac{1}{X}\int_o^X i_{xr} \cdot dx = \frac{1}{X} \cdot \frac{i_{Xr}}{\cosh \alpha X} \cdot \int_o^X \cosh \alpha x \cdot dx$$
$$= \frac{1}{\alpha X} \cdot \frac{i_{Xr}}{\cosh \alpha X} \cdot \sinh \alpha X = i_{Xr} \frac{\tanh \alpha X}{\alpha X}$$
$$\frac{\text{absamperes}}{\text{sq. cm.}} \angle \qquad (30)$$

Hence the skin-effect impedance ratio $\frac{Z}{R}$ of the whole strip is

$$\frac{Z}{R} = \frac{i_{Xr}}{i_{qr}} = \frac{\alpha X}{\tanh \alpha X} \qquad \text{numeric} \angle \qquad (31)$$

The real component of this expression is the resistance ratio R′/R, the imaginary component is the reactance ratio $\frac{L'\omega}{R}$

The quantity $\frac{\tanh \alpha X}{\alpha X}$ where αX is a semi-imaginary, has been tabulated and charted as far as the modulus of $\alpha X = 3.0$ and $\tanh \alpha X$ of the same semi-imaginary to a modulus of 20.

The solution of (25) is also

$$\mathfrak{B}_x = A_2 \cosh \alpha x + B_2 \sinh \alpha x \qquad \text{gausses} \angle \qquad (32)$$

and at $\alpha x = 0$ $\qquad \mathfrak{B}_o = A_2 = 0 \qquad \text{gausses} \angle \qquad (33)$

Consequently $\qquad \mathfrak{B}_x = B_2 \sinh \alpha x \qquad \text{gausses} \angle \qquad (34)$

and at X $\qquad \mathfrak{B}_X = \frac{4\pi I}{b} \qquad \text{gausses} \angle \qquad (35)$

whence $\qquad B_2 = \frac{4\pi I}{b \sinh \alpha X} \qquad \text{gausses} \angle \qquad (36)$

so that $\qquad \mathfrak{B}_x = \frac{4\pi I}{b} \cdot \frac{\sinh \alpha x}{\sinh \alpha X} \qquad \text{gausses} \angle \qquad (37)$

The magnetic flux density in the strip, substantially in horizontal-layer distribution, is thus directly proportional, at any distance x from the outside surface, to the hyperbolic sine of the position angle αx. Such sines have all been tabulated and charted.

When the position angle αX at the inner surface of the strip exceeds in modulus 6.0, tanh αX is very nearly unity, and the skin-effect impedance ratio $\frac{Z}{R}$ becomes αX simply, a semi-imaginary. The real component, or skin-effect resistance-ratio, then becomes $R'/R = |\alpha X|/\sqrt{2} = \alpha_2 X$, and is equal to the imaginary component or skin-effect reactance ratio $L'\omega/R$, while the equivalent skin depth δ is $1/\alpha_2$ centimeter.

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APPENDIX IV

ELEMENTARY THEORY OF THE LINEAR IMPEDANCE OF A LONG STRAIGHT CONDUCTOR FORMED OF SEVEN EQUALLY SPACED STRANDS UNSPIRALLED

Let a central round wire O Figure 31, of radius ρ centimeters be surrounded by six similar and parallel wires 1 to 6, forming a hexagon all at equal interaxial distances d centimeters. The linear resistance of each wire, with its own extra skin-effect resistance is r absohms per linear centimeter. The return con-

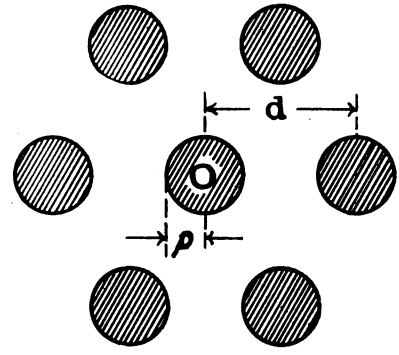

FIGURE 31—Spacing of 7-Strand Conductor.

ductor is a similar parallel open strand, at a distance D centimeters, which is large with respect to d. A simple alternating e. m. f. of angular velocity $\omega = 2\pi f$ radians per second, is impressed upon the loop. Required the linear impedance of the conductor.

Let i_o be the r. m. s. complex current strength in the middle wire (absamperes $\angle$)

i_1 be the r. m. s. complex current strength in any outside wire (absamperes $\angle$)

l_o be the linear inductance of the middle wire (abhenrys/cm.)

l_1 be the linear inductance of any outside wire (abhenrys/cm.)

z_o be the linear impedance of the middle wire (absohms/cm. $\angle$)

z_1 be the linear impedance of any outside wire (absohms/cm. $\angle$)

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m be the linear mutual inductance between middle and an outside wire (abhenrys/cm. $\angle$)

m' be the linear mutual inductance of any five outer wires on the sixth (abhenrys/cm. $\angle$)

M be the linear mutual inductance of return strand on any wire (abhenrys/cm. $\angle$)

e be the linear r. m. s. e. m. f. impressed on any wire $\left(\frac{\text{abvolts}}{\text{cm.}} \angle\right)$

$\varepsilon = 2.71828 \cdots$, $j = \sqrt{-1}$.

Then on any outside wire

$$e = i_1 r + j\, i_1 l_1 \omega + j\, i_o m \omega + j\, 5\, i_1 m' \omega - j\, 6\, i_1 M \omega - j\, i_o M \omega \quad \frac{\text{abvolts}}{\text{cm.}} \angle \qquad (38)$$

and on the inside wire

$$e = i_o r + j\, i_o l_o \omega + j 6 i_1 m \omega - j 6 i_1 M \omega - j\, i_o M \omega \quad \frac{\text{abvolts}}{\text{cm.}} \angle \qquad (39)$$

The values of the self-inductances are

$$l_o = l_1 = 2\left(logh \frac{2x}{\varepsilon \rho} + \frac{\mu}{4}\right) \quad \frac{\text{abhenrys}}{\text{cm.}} \angle \qquad (40)$$

where x is the length of the conductor, *logh.* represents hyperbolic or Napierian logarithms, and μ is the permeability of the wire which for copper may be taken as unity.

The values of the mutual inductances are

$$m = 2 logh \left(\frac{2x}{\varepsilon d}\right) \quad \frac{\text{abhenrys}}{\text{cm.}} \qquad (41)$$

$$M = 2 logh \left(\frac{2x}{\varepsilon D}\right) \quad \frac{\text{abhenrys}}{\text{cm.}} \qquad (42)$$

$$m' = 2 logh \left(\frac{2x}{\varepsilon d\, 6^{\frac{1}{5}}}\right) \quad \frac{\text{abhenrys}}{\text{cm.}} \qquad (43)$$

Since the geometrical mean distance of all five outer wires from the sixth is

$$\sqrt[5]{d \cdot \sqrt{3}\, d \cdot 2d \cdot \sqrt{3}\, d \cdot d} = d \sqrt[5]{6} \qquad (44)$$

Equating (38) and (39) we have

$$i_1 \{r + j\omega (l_1 + 5m' - 6M)\} + i_o \{j\omega (m - M)\} = i_1 \{j\omega (6m - 6M)\} + i_o \{r + j\omega (l_o - M)\} \quad \text{abvolts} \angle \qquad (45)$$

or

$$i_1 \left\{r + 2j\omega \left(logh \frac{2x}{\varepsilon \rho} + \frac{\mu}{4} + 5\, logh \frac{2x}{\varepsilon d\, 6^{\frac{1}{5}}} - 6\, logh \frac{2x}{\varepsilon d}\right)\right\} = i_o \left\{r + 2j\omega \left(logh \frac{2x}{\varepsilon \rho} + \frac{\mu}{4} - logh \frac{2x}{\varepsilon d}\right)\right\} \quad \text{abvolts} \angle \qquad (46)$$

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whence

$$i_o = k\, i_1 \quad \text{where } k = \frac{r + 2\,j\,\omega \left\{ \log_h \left(\frac{d}{6\,\rho} \right) + \frac{\mu}{4} \right\}}{r + 2\,j\,\omega \left\{ \log_h \left(\frac{d}{\rho} \right) + \frac{\mu}{4} \right\}} \quad \text{numeric} \angle \qquad (47)$$

$$z_1 = r + 2\,j\,\omega \left\{ \log_h \left(\frac{D^6}{6\,d^5\,\rho} \right) + \frac{\mu}{4} + k \log_h \left(\frac{D}{d} \right) \right\} \quad \frac{\text{absohms}}{\text{cm.}} \angle \qquad (48)$$

$$z_o = r + 2\,j\,\omega \left\{ \log_h \left(\frac{D}{\rho} \right) + \frac{\mu}{4} + \frac{6}{k} \log_h \left(\frac{D}{d} \right) \right\} \quad \frac{\text{absohms}}{\text{cm.}} \angle \qquad (49)$$

and

$$Z = \frac{1}{\frac{6}{z_1} + \frac{1}{z_o}} \quad \frac{\text{absohms}}{\text{cm.}} \angle \qquad (50)$$

Taking the case of the test represented in Figure 10, with $f = 50{,}000^{\sim}$; or $\omega = 314{,}159$, $\rho = 0.03213$ centimeter, $d = 20\,\rho$, or 10 diameters $= 0.643$ centimeter, $D = 30\,d = 300\,\rho = 9.64$ centimeters, $r = 5.711 \times 10^5$ absohms per linear centimeter, which includes a skin-effect resistance ratio of 1.030 for this wire remote from other conductors, we find $k = 0.50872 \angle -(16^\circ\, 22')$. That is, the current strength in the middle wire is only a little more than half that on any one of the six outer wires, and lags in phase 16° 22′ relatively thereto. Substituting in (48), we find $z_1 = 11.981 \times 10^6 \angle 86^\circ\, 05'\, 58''$ absohms per centimeter and $z_o = 23.551 \times 10^6 \angle 102^\circ\, 27'\, 58''$. The joint linear impedance of the 7 strands is then $Z = 1.8461 \times 10^6 \angle 87^\circ\, 21'\, 55''$ absohms per centimeter $= (0.084865 + j\, 1.8461)\, 10^6 = R' + j\, X'$. Since the d. c. linear resistance of the same strands would be $R = 0.07920 \times 10^6$ absohms per centimeter, the skin-effect resistance ratio $R'/R = 1.0715$. The observed resistance-ratio of this 7-strand conductor at 10 diameter spacing and $50{,}000^{\sim}$ was observed to be 1.072, which is in satisfactory agreement. At closer spacings, however, the agreement does not appear to be so good, the observed resistance-ratios being greater than the computed. Thus, at $d = 10\,\rho$, the computed ratio is 1.08, but the observed was 1.11. It may be that at the closer spacings the current over the cross-sections of the wires is further distorted by proximity effect, thereby increasing the power loss and resistance ratio.

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LIST OF SYMBOLS EMPLOYED

A_1, B_1 Arbitrary constants of current density (absamperes/sq. cm. $\angle$)

A_2, B_2 Arbitrary constants of flux density (gauses $\angle$)

$a_o = \sqrt{-j\,4\pi\gamma\mu\omega} = a_2 - j\,a_2$, semi-imaginary propagation constant in round wires (cm.$^{-1}$ $\angle$)

$a = \sqrt{j\,4\pi\gamma\mu\omega} = a_2 + j\,a_2$, semi-imaginary propagation constant in sheets (cm.$^{-1}$ $\angle$)

a_2 Components of a_o or a. (cm.$^{-1}$)

$\mathfrak{B}$ Magnetic flux density (gausses)

$\mathfrak{B}_o$ Magnetic flux density at external surface of strip $x = 0$ (gausses $\angle$)

$\mathfrak{B}_x$ Magnetic flux density at distance x (gausses $\angle$)

$\mathfrak{B}_X$ Magnetic flux density at distance X (gausses $\angle$)

b Width of a flat strip (cm.)

β° Argument of a complex quantity (degrees)

$\gamma = 1/\rho$ Electric conductivity of a metal (abmhos/cm.)

D Distance of return conductor from going conductor (cm.)

d Thickness of an insulating layer between two strips (cm.)

Also interaxial distance between strands in a 7-strand conductor (cm.)

δ Depth of equivalent skin (cm.)

e Linear alternating e. m. f. impressed upon one wire of a strand (r. m. s. abvolts per cm. $\angle$)

$\varepsilon = 2.71828 \ldots$ the Naperian base

f Impressed frequency (cycles per second)

$\mathfrak{H}$ Magnetic intensity (gilberts/cm.)

i_{qr} Root-mean-square mean vector current density in a conducting strip (absamperes/sq. cm. $\angle$)

i_x Instantaneous current strength at distance x from origin $\left(\frac{\text{absampere}}{\text{cm.}^2}\ \angle\right)$

i_{xm} Maximum cyclic current strength at distance x from origin $\left(\frac{\text{absampere}}{\text{cm.}^2}\ \angle\right)$

i_{xr} Root-mean-square current strength at distance x from origin $\left(\frac{\text{absampere}}{\text{cm.}^2}\ \angle\right)$

i_o Root-mean-square current strength in central wire of a strand $\left(\frac{\text{absampere}}{\text{cm.}^2}\ \angle\right)$

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i_1 Root-mean-square current strength in external wire of a strand $\left(\frac{\text{absampere}}{\text{cm.}^2} \angle\right)$

$j = \sqrt{-1}$

$J_o(\ )$ Bessel function of zero order (numeric $\angle$)

$J_1(\ )$ Bessel function of first order (numeric $\angle$)

k Ratio of current strength in central strand to current strength in an external strand (numeric $\angle$)

l Length of a wire of resistance R_o or R'_o ohms (cm.)

l_o Linear self inductance of central wire in a strand (abhenrys/cm.)

l_1 Linear self inductance of external wire in a strand (abhenrys/cm.)

M The modulus of the impedance ratio Z/R (numeric) Also the linear mutual inductance of return conductor on going conductor (abhenrys/linear cm. $\angle$)

m_o Linear mutual inductance between middle and any outside wire (abhenrys/linear cm. $\angle$)

m' Linear mutual inductance of any five outer wires of 7-strand conductor on the sixth (abhenrys/linear cm.)

μ Permeability (gausses/gilberts per cm.)

$\pi = 3.14159\ldots$

R Linear resistance of a conductor to continuous currents (ohms or absohms/cm.)

R' Linear resistance of a conductor to alternating currents (ohms or absohms/cm.)

R_o Linear resistance of a conductor to continuous currents (ohms per meter)

R'_o Linear resistance of a conductor to continuous currents (ohms per 1,000 feet)

$\rho = 1/\gamma$ Resistivity of conductor (absohm-cm.) Also radius of central wire in a 7-strand conductor (cm.)

X' Linear reactance of a conductor in the presence of skin-effect $\left(\frac{\text{absohms}}{\text{cm.}} \angle\right)$

X Radius of a round wire. Also thickness or half-thickness of a strip (cm.)

x Radius of a point in the cross-section of a wire (cm.)

z_o Linear impedance of central strand in a 7-strand conductor (absohms/cm $\angle$).

z_1 Linear impedance of any external strand in a 7-strand conductor (absohms/cm. $\angle$

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Z Linear impedance of a straight conductor in the presence of skin-effect $\left(\frac{\text{absohms}}{\text{cm.}} \angle\right)$

Also linear impedance of a 7-strand conductor (absohms/cm. $\angle$)

$\omega = 2\pi f$ Impressed angular velocity (radians/sec.)

$\sim$ Sign for cycles per second

μf. Sign for microfarad

mμf. Sign for millimicrofarad

$\angle$ Sign of a complex quantity or plane vector

SUMMARY: After defining the skin-effect resistance and reactance ratios, the "spirality" and "proximity effects" for conductors at radio-frequencies, a differential bridge circuit supplied with current from 10,000 to 100,000 cycles by an Alexanderson alternator is described.

There are tested at various frequencies straight solid wires, stranded wires (of various spiralities, and also braided), wires with definitely spaced strands, very finely stranded wire (with insulated strands), braided "litzendraht" wire, stranded twisted wire (of various spiralities), strips (both singly and in opposed proximity), and tubes.

Important practical design data, quantitative as well as comparative, are given for the range of frequencies investigated.

In a number of appendices to the paper, the details of the apparatus and certain theoretical calculations of radio-frequency resistance are given.

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LIST OF RESEARCH BULLETINS

Bulletin Number

*1. The Economical Transportation of Merchandise in Metropolitan Districts. H. Pender and H. F. Thomson; March, 1912.

*2. Notes on the Cost of Motor Trucking. H. Pender and H. F. Thomson; October, 1912.

3. Observations on Horse and Motor Trucking. H. Pender and H. F. Thomson; March, 1913.

4. Relative Fields of Horse, Electric and Gasoline Trucks. H. F. Thomson; August, 1914.

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13. Skin-Effect Resistance Measurements of Conductors at Radio-Frequencies up to 100 000 Cycles per Second. A. E. Kennelly and H. A. Affel; December, 1916.

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